Eco 202

Prof. Smith

Additional Problems for Review One

Note: Please treat x_1 as x and x_2 as y!

1

i) Uncompensated Demand: $x_1 = \frac{1}{3} \cdot \frac{I}{P_1}$ $\quad x_2 = \frac{2}{3} \cdot \frac{I}{P_2}$

ii) Compensated Demand:

$$\mathcal{L} = P_1 x_1 + P_2 x_2 - \lambda (x_1 x_2^2 - u)$$

$$\left.\begin{aligned} \mathcal{L}_{x_1} &= P_1 - \lambda x_2^2 = 0 \\ \mathcal{L}_{x_2} &= P_2 - \lambda 2 x_1 x_2 = 0 \end{aligned}\right\} \quad \frac{P_1}{x_2^2} = \lambda = \frac{P_2}{2 x_1 x_2}$$

$$\mathcal{L}_\lambda = -(x_1 x_2^2 - u) = 0$$

$$\boxed{\frac{P_1}{P_2} = \frac{x_2}{2 x_1}} \quad (1)$$

$$(2) \quad \boxed{x_1 x_2^2 = u}$$

$(1) \rightarrow (2)$

$$x_1 = \frac{x_2}{2} \cdot \frac{P_2}{P_1}$$

$$\left(\frac{x_2}{2} \cdot \frac{P_2}{P_1}\right) \cdot x_2^2 = u$$

$$x_2^3 = \frac{P_1}{P_2} \cdot 2u$$

$$\boxed{x_2 = \left(\frac{P_1}{P_2} \cdot 2u\right)^{1/3}}$$

$$x_2 = 2 x_1 \cdot \frac{P_1}{P_2}$$

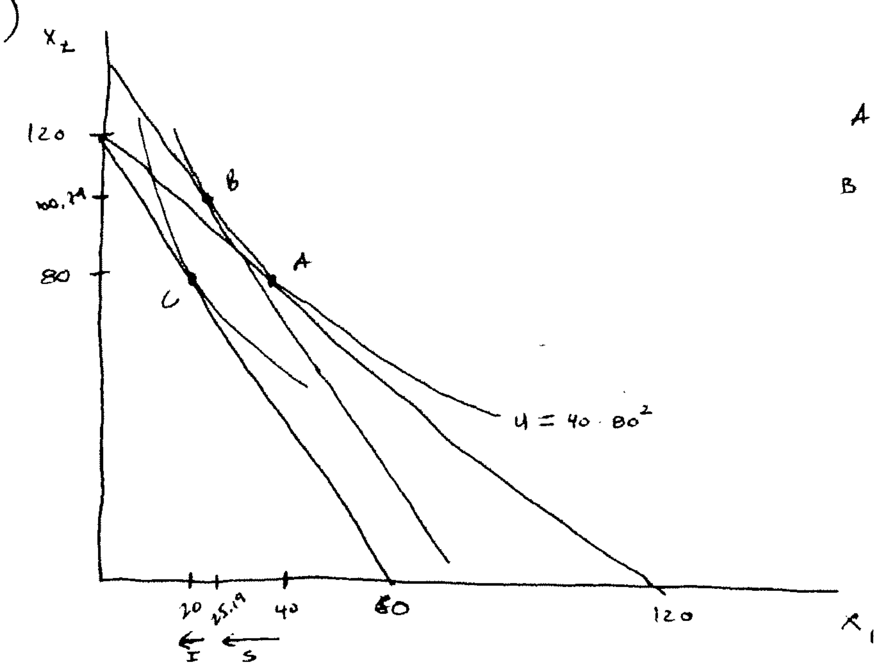
$$x_1 \cdot \left(2 x_1 \frac{P_1}{P_2}\right)^2 = u$$

$$x_1 \cdot 4 \cdot x_1^2 \cdot \left(\frac{P_1}{P_2}\right)^2 = u$$

$$x_1^3 = \frac{u}{4} \cdot \frac{P_2^2}{P_1^2}$$

$$\boxed{x_1 = \left(\frac{u}{4} \cdot \frac{P_2^2}{P_1^2}\right)^{1/3}}$$

iii)



A to B: Substitution

B to C: Income

A: $x_1 = \frac{1}{3} \cdot \frac{120}{1} = 40$

$x_2 = \frac{2}{3} \cdot \frac{120}{1} = 80$

B: use compensated demands

$$x_1 = \left(\frac{(40 \cdot 80^2)}{4} \cdot \frac{1}{4}\right)^{1/3} = \left(\frac{256{,}000}{16}\right)^{1/3}$$

$x_1 = 25.19$

$$x_2 = \left(\frac{2}{1} \cdot 2(40 \cdot 80^2)\right)^{1/3}$$

$x_2 = 100.79$

C: $x_1 = \frac{1}{3} \cdot \frac{120}{2} = 20$

$x_2 = \frac{2}{3} \cdot \frac{120}{1} = 80$

iv)

$$\frac{dx_1}{dP_1} = \frac{dx_1^{comp}}{dP_1} - x_1 \cdot \frac{dx_1}{dI}$$

use initial price!

Total: $\frac{dx_1}{dP_1} = -\frac{1}{3} \cdot \frac{I}{P_1^2} = -\frac{120}{3 \cdot 1^2} = -40$

Income: $\begin{cases} x_1 = 40 \\ \frac{dx_1}{dI} = \frac{1}{3} \cdot \frac{1}{P_1} = \frac{1}{3} \end{cases}$

$\rightarrow -40 * \frac{1}{3}$

$$-40 = \frac{dx_1^{comp}}{dP_1} - 40 \cdot \frac{1}{3}$$

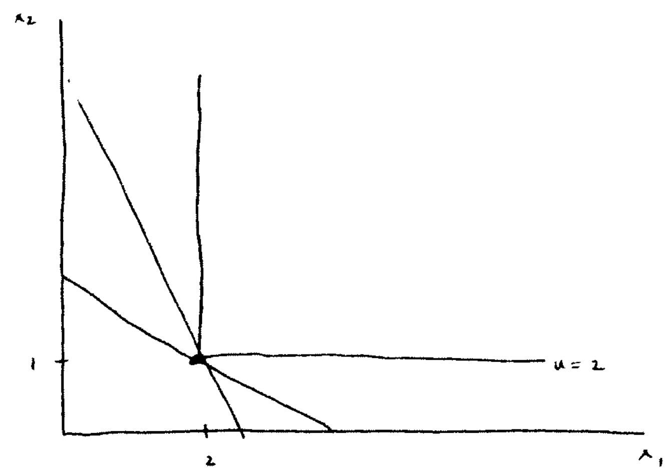
$\frac{dx_1^{comp}}{dP_1}$ = substitution Effect $= -26.67$

$\boxed{2}$ $u = \min\{x_1, 2x_2\}$

i) $x_1 = 2x_2 \rightarrow P_1 x_1 + P_2 x_1/2 = I$ $\boxed{x_1 = \frac{I}{P_1 + P_2/2}}$

$\rightarrow P_1 2x_2 + P_2 x_2 = I$ $\boxed{x_2 = \frac{I}{2P_1 + P_2}}$

ii)

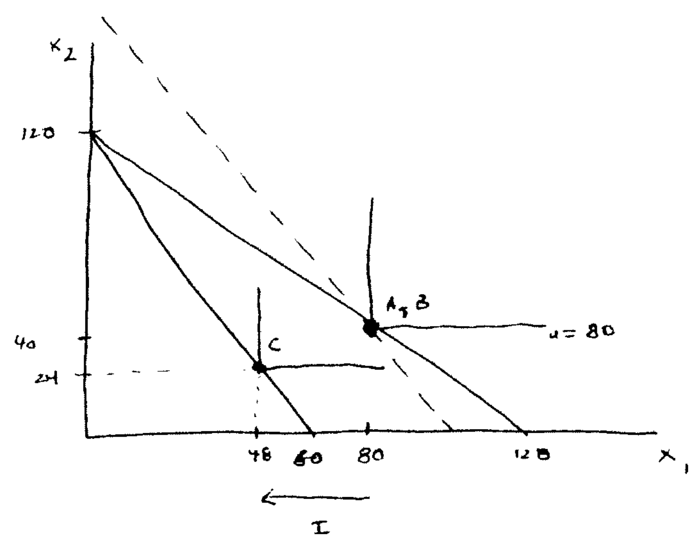


Idea: Minimize Expenditure subject to a utility target.

Example → Regardless of Prices consume at corner $x_1 = 2$ $x_2 = 1$

$\boxed{x_1 = u \qquad x_2 = \frac{u}{2}}$ → Compensated Demand

iii)



A: $P_1 = 1 = P_2$ $I = 120$

$x_1 = 80$

$x_2 = 40$

B: Old utility, New Prices

→ Use comp demand

$x_1 = 80$ $x_2 = 40$

C: $P_1 = 2$ $P_2 = 1$ $I = 120$

$x_1 = 48$ $x_2 = 24$

A to B : Substitution

B to C : Income

No substitution effect!

iv)

$$\frac{dx_1}{dP_1} = \frac{dx_1^{comp}}{dP_1} - x_1 \frac{dx_1}{dI}$$

Total $\frac{dx_1}{dP_1} = -\frac{I}{\left(P_1 + \frac{P_2}{2}\right)^2} = -\frac{120}{\left(\frac{3}{2}\right)^2} = -53.3$

Income:

$$\frac{dx_1}{dI} = \frac{1}{P_1 + P_2/2} = \frac{2}{3}$$

$$x_1 = 80$$

$$\longrightarrow \quad -80 \cdot \frac{2}{3} = -53.3$$

$$-53.3 = \frac{dx_1^{comp}}{dP_1} - 53.3$$

$$\frac{dx_1^{comp}}{dP_1} = 0$$

3

i) $\boxed{x_1 = \frac{P_2}{P_1} \qquad x_2 = \frac{I}{P_2} - 1}$

ii) $\mathcal{L} = P_1 x_1 + P_2 x_2 - \lambda(\ln x_1 + x_2 - u)$

$$\left.\begin{aligned} \mathcal{L}_{x_1} &= P_1 - \lambda \frac{1}{x_1} = 0 \\ \mathcal{L}_{x_2} &= P_2 - \lambda = 0 \end{aligned}\right\} \quad P_1 x_1 = P_2 \qquad x_1 = \frac{P_2}{P_1}$$

$$\mathcal{L}_\lambda = -(\ln x_1 + x_2 - u) = 0$$

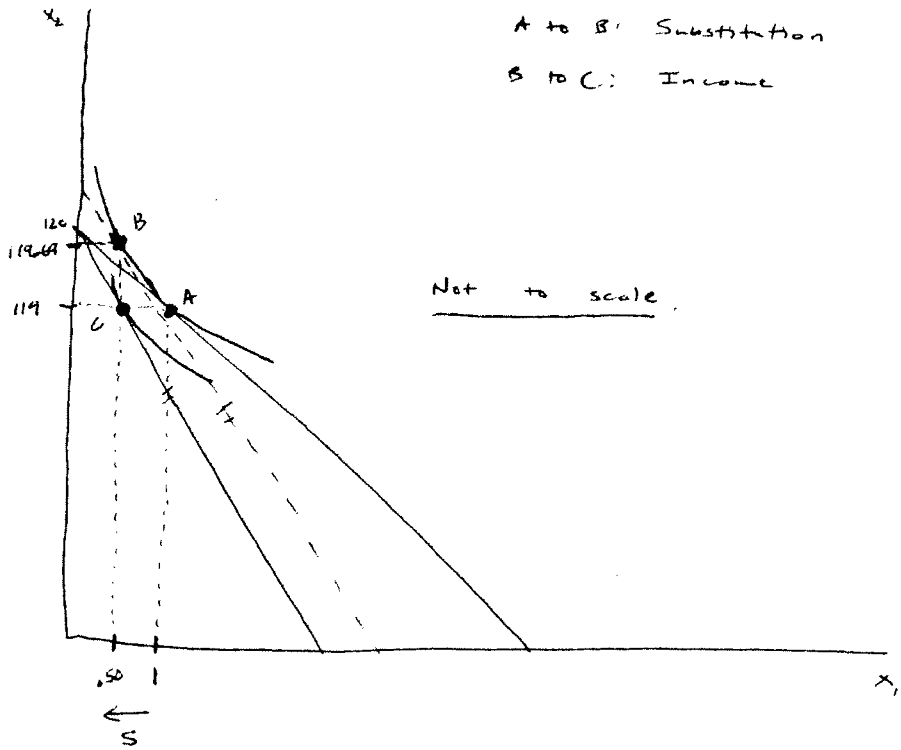
$$\longrightarrow \ln x_1 + x_2 = u$$

$$\ln\left(\frac{P_2}{P_1}\right) + x_2 = u$$

$$\boxed{x_2 = u - \ln \frac{P_2}{P_1}} \qquad \boxed{x_1 = \frac{P_2}{P_1}} \quad \rightarrow$$

Compensated Demand Functions

iii)



A to B: Substitution

B to C: Income

Not to scale.

<u>A:</u> $P_1 = P_2 = 1$
$I = 120$

$x_1 = 1$

$x_2 = 119$

<u>B:</u> $P_1 = 2 \quad P_1 = 1$
$I = 120$

$x_1 = .50$

$x_2 = 119 - \ln(.50)$

$x_2 = 119.69$

<u>C:</u> $P_1 = 2 \quad P_1 = 1$

$x_1 = .50$

$x_2 = 119$

iv)

$$\frac{dx_1}{dP_1} = \frac{dx_1^{comp}}{dP_1} - x_1 \frac{dx_1}{dI} \qquad -1 = \frac{dx_1^{comp}}{dP_1} - 1 \cdot 0$$

Total: $$\frac{dx_1}{dP_1} = -\frac{P_2}{P_1^2} = -\frac{1}{1^2} = -1 \qquad \frac{dx_1^{comp}}{dP_1} = \text{Subs.} = -1$$

$$X_1 = \frac{P_2}{P_1} = 1$$

Income:

$$\frac{dx_1}{dI} = 0$$

$$-1 \cdot 0 = 0$$

$\boxed{4}$

i) $x_1 = \begin{cases} 0 & \frac{p_1}{p_2} > MRS \\ 0,\ I/p_1 & \frac{p_1}{p_2} = MRS \\ \frac{I}{p_1} & \frac{p_1}{p_2} < MRS \end{cases}$ $\qquad x_2 = \begin{cases} 0 & \frac{p_1}{p_2} < MRS \\ [0,\ I/p_2] & \frac{p_1}{p_2} = MRS \\ I/p_2 & \frac{p_1}{p_2} > MRS \end{cases}$

$(MRS = \frac{3}{2})$

ii) Use graphical analysis

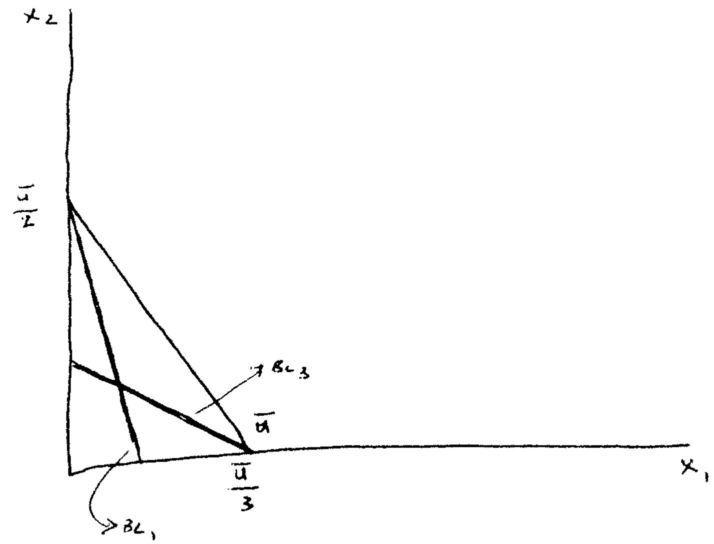
let $\bar{u} = 3x_1 + 2x_2$

if $x_2 = 0$

$x_1 = \frac{\bar{u}}{3}$

if $x_1 = 0$

$x_2 = \frac{\bar{u}}{2}$



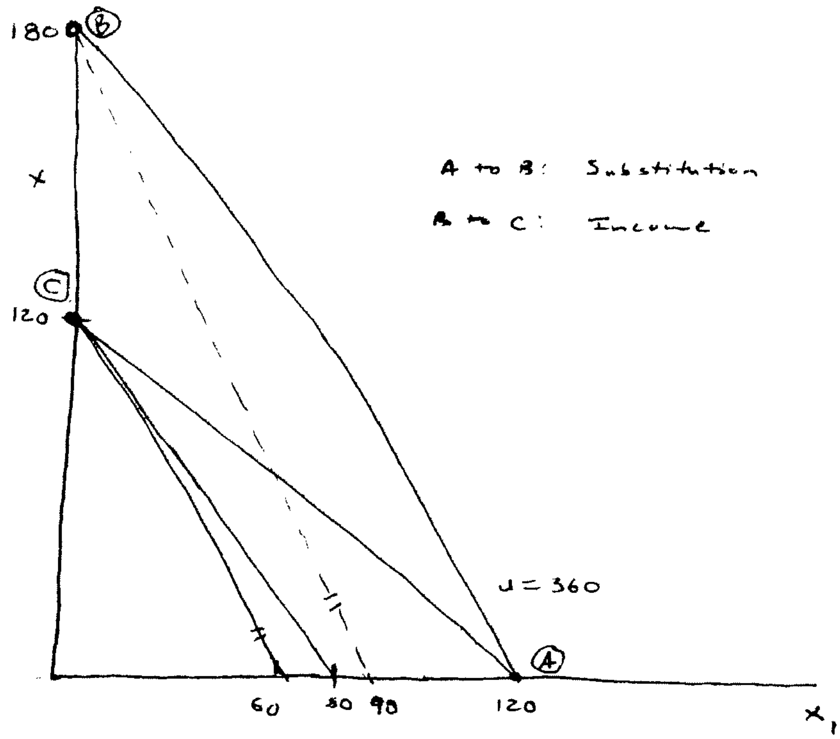
Case 1 : If $\frac{p_1}{p_2} > MRS$, buy only x_2.
(BL_1)

Case 2 : If $\frac{p_1}{p_2} = MRS$, buy any bundle on $\bar{u}$.

Case 3 : If $\frac{p_1}{p_2} < MRS$, buy only x_1.
(BL_3)

$$x_1^{comp} = \begin{cases} 0 & \text{if } \frac{p_1}{p_2} > MRS \\ (0,\ u/3) & \text{if } \frac{p_1}{p_2} = MRS \\ \frac{u}{3} & \text{if } \frac{p_1}{p_2} < MRS \end{cases} \qquad x_2^{comp} = \begin{cases} 0 & \text{if } \frac{p_1}{p_2} < MRS \\ (0,\ u/2) & \text{if } \frac{p_1}{p_2} = MR \\ \frac{u}{2} & \text{if } \frac{p_1}{p_2} > MRS \end{cases}$$

iii)



A to B: Substitution

B to C: Income

$\underline{A}$: $\frac{P_1}{P_2} = 1$ $\quad I = 120$

$x_1 = 120$

$x_2 = 0$

$\underline{B}$: $\frac{P_1}{P_2} = 2$

$x_1 = 0$

$x_2 = 180$

$\underline{C}$: $\frac{P_1}{P_2} = 2$ $\quad I = 120$

$x_1 = 0$

$x_2 = 120$

iv) Since initial values are $P_x = P_y = 1$ and $I = 120$ the relevant portion of the demand function is

$$X = \frac{I}{P_x}$$

$$\frac{dX}{dP_x} = \frac{dX^c}{dP_x} - X \cdot \frac{dX}{dI}$$

$$\frac{dX}{dP_x} = \frac{-I}{P_x^2} \qquad -\frac{120}{1^2} = \frac{dX^c}{dP_x} - \frac{120}{1} \cdot \left(\frac{1}{1}\right)$$

$$\frac{dX}{dI} = \frac{1}{P_x} \qquad \text{Subs} = \frac{dX^c}{dP_x} = -120 + 120 = \phi$$

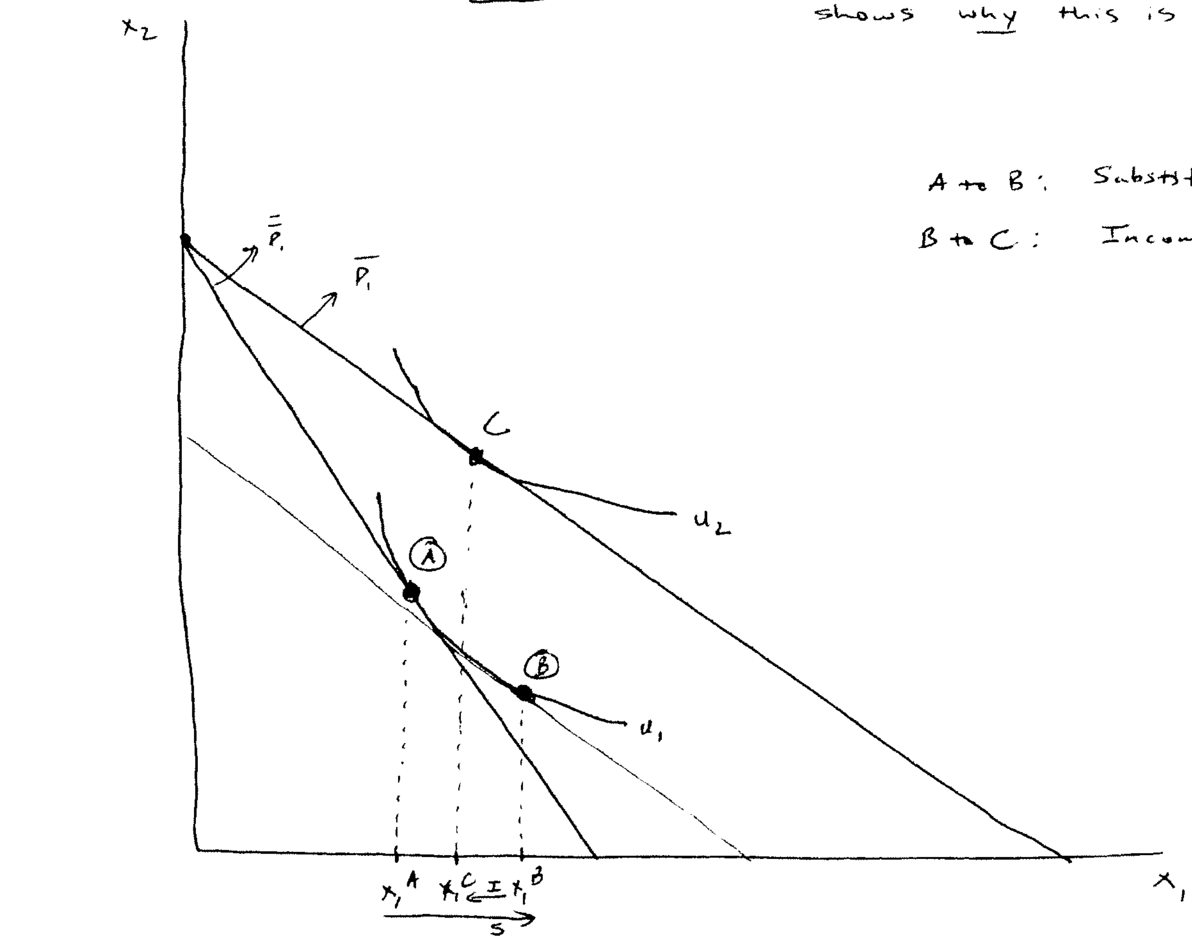
Total Effect = -120

Income Effect = -120

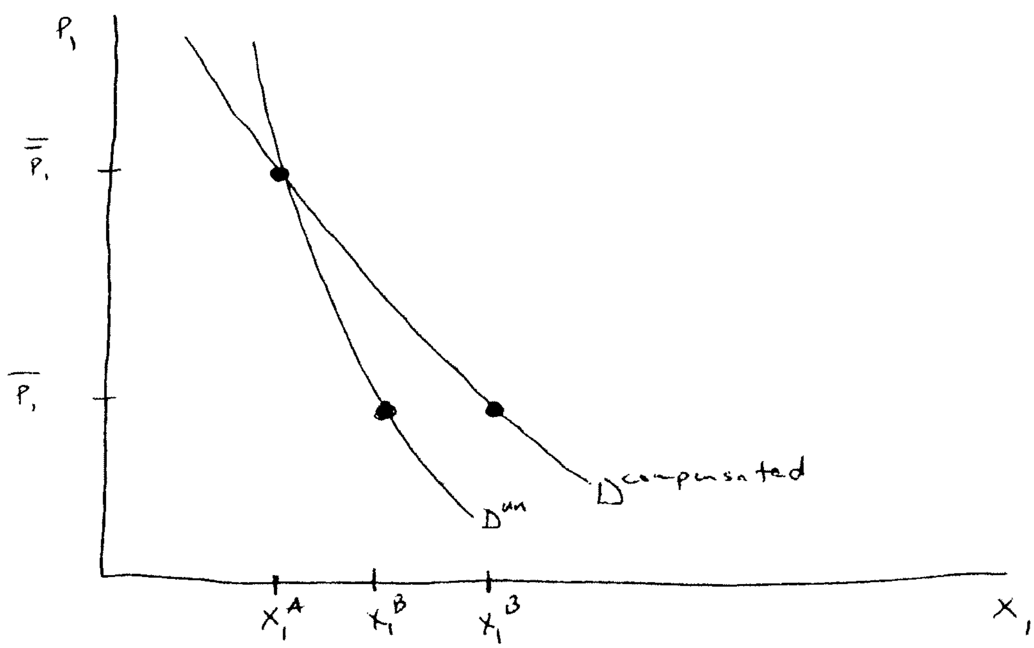
II

1

$\boxed{\text{True}}$ → Graphical Analysis shows why this is true.

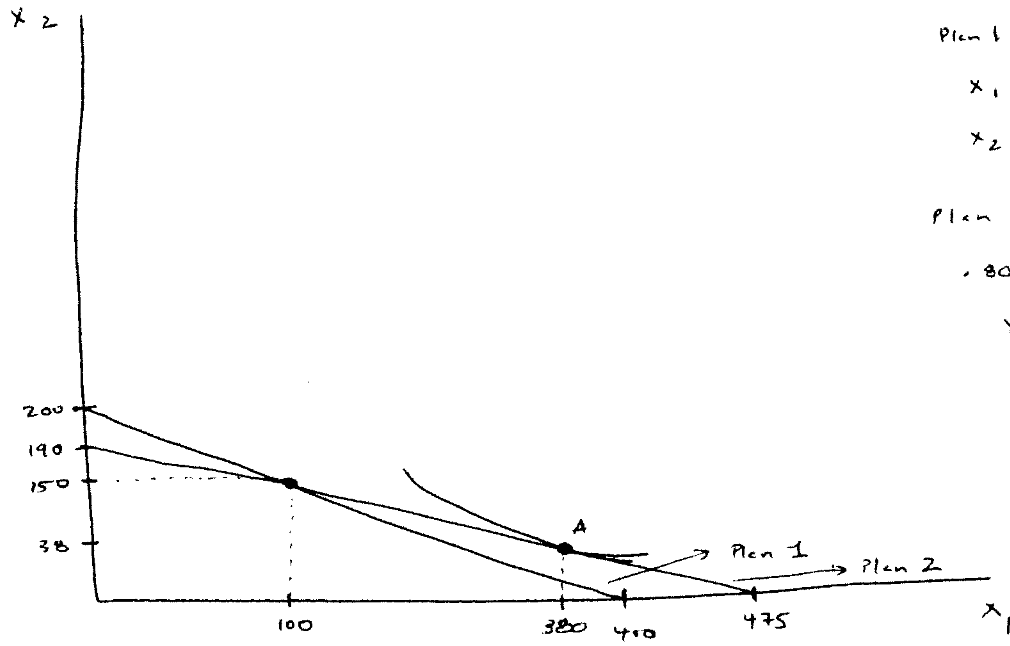


→ x_1 is inferior, P_1 falls.



II

$\boxed{2}$



Plan 1

$x_1 + 2x_2 = 400$

$x_2 = 200 - \frac{1}{2} x_1$

Plan 2

$.80 x_1 + 2x_2 = 380$

$x_2 = 190 - .40 x_1$

a) $x_1 = \frac{4}{5} \cdot \frac{I}{P_1}$

$x_2 = \frac{1}{5} \cdot \frac{I}{P_2}$

} will choose plan 2.

$2 \rightarrow u = 380^4 \cdot 38$

$1 \rightarrow u = 320^4 \cdot 40$

b) $x_1 = \frac{1}{2} \cdot \frac{I}{P_1} + \frac{P_2}{2P_1}$

$x_2 = \frac{1}{2} \cdot \frac{I}{P_2} - \frac{1}{2}$

Plan 1 $\Rightarrow$ $x_1 = 201$, $x_2 = 99.5$

Plan 2 $\Rightarrow$ $x_1 = 238.75$, $x_2 = 94.5$ } Choose Plan 2.

c) Need to consume at $x_1 = 100$ $x_2 = 150$

$x_2 = 1.5 x_1 \Rightarrow 2x_2 = 3x_1$

$U = \text{Min}\{3x_1, 2x_2\}$ $\rightarrow$ $U = \text{Min}\{ax_1, bx_2\}$

$\downarrow$

Any answer where $a = \frac{3}{2} b$!

II

$\boxed{3}$

a) $x_1 = \frac{1}{4} \cdot \frac{I}{P_1}$ $\qquad x_2 = \frac{3}{4} \cdot \frac{I}{P_2}$

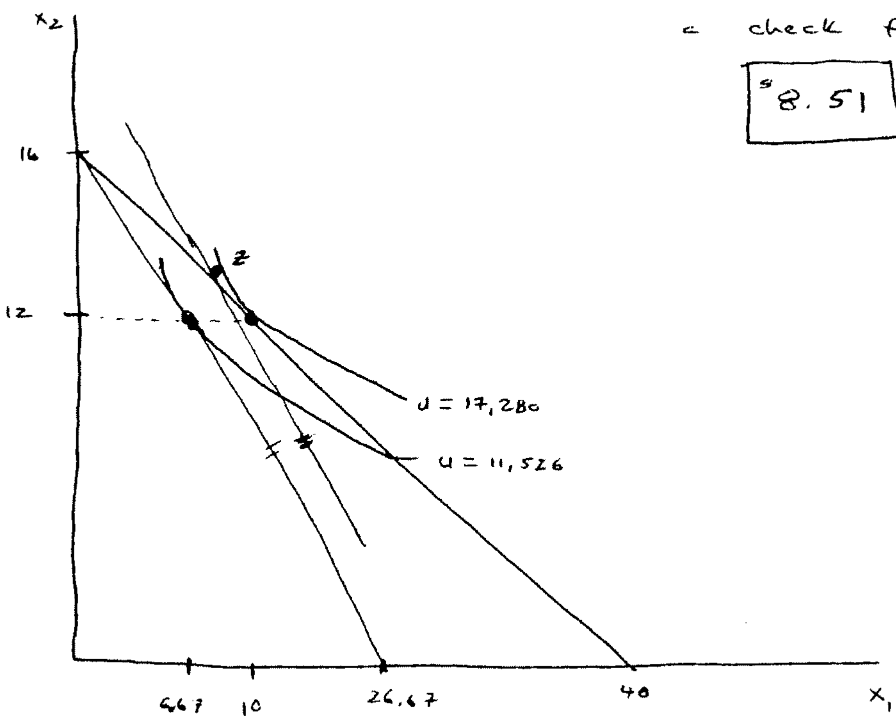
$x_1^* = \frac{1}{4} \cdot \frac{80}{2} = 10$ $\qquad x_2^* = \frac{3}{4} \cdot \frac{80}{5} = 12$

b) $P_1 = \$2 + t = \$3$

$x_1^* = \frac{80}{4 \cdot 3} = 6.67$ $\qquad x_2^* = 12$

c)

→ Must give each family a check for $\boxed{\$8.51}$



Idea: Find how much bundle Z would cost.

$$\mathcal{L} = 3x_1 + 5x_2 - \lambda(x_1 x_2^3 - 17{,}280)$$

$$\mathcal{L}_{x_1} = 3 - \lambda x_2^3 = 0 \qquad \frac{3}{x_2^3} = \frac{5}{3x_1x_2^2} \qquad 9x_1x_2^2 = 5x_2^3$$

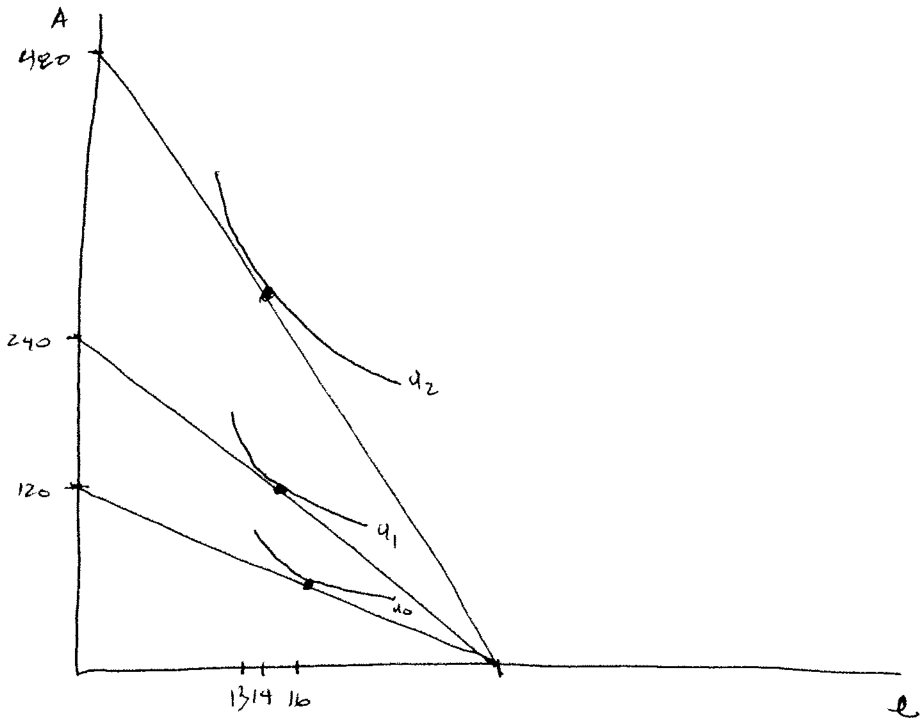
$$\mathcal{L}_{x_2} = 5 - \lambda 3x_1x_2^2 = 0 \qquad 9x_1 = 5x_2$$

$$\mathcal{L}_{\lambda} = -(x_1x_2^3 - 17{,}280) = 0 \qquad x_1 = 5/9\, x_2$$

$$\frac{5}{9}x_2 \cdot x_2^3 = 17{,}280 \qquad x_2^* = 13.28 \quad x_1^* = 7.37 \quad \Rightarrow 3(7.37) + 5(13.28) = \$88.51$$

II

$\boxed{4}$



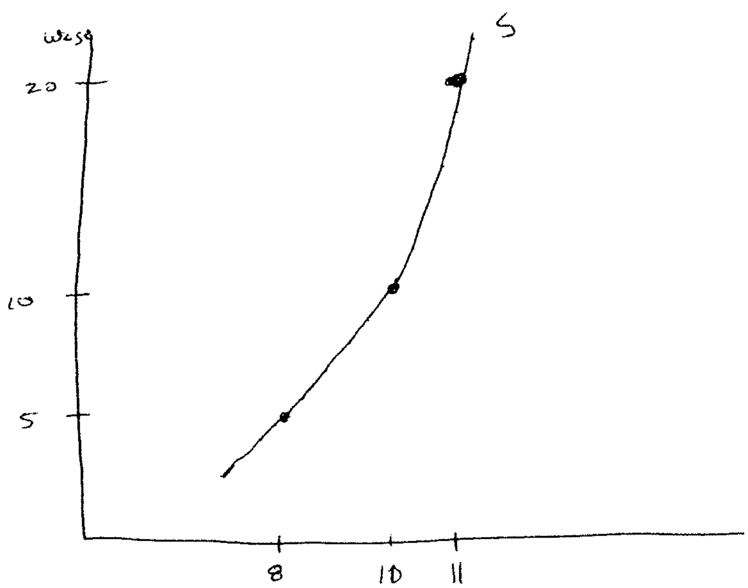
$$w \cdot \ell + A = w \cdot 24$$

$\bar{w} = 5$

$\bar{\bar{w}} = 10$

$\bar{\bar{\bar{w}}} = 20$

Labor = $24 - \ell$



<u>This</u> labor supply <u>doesn't</u> bend backwards. The income effect never dominates the substitution effect.