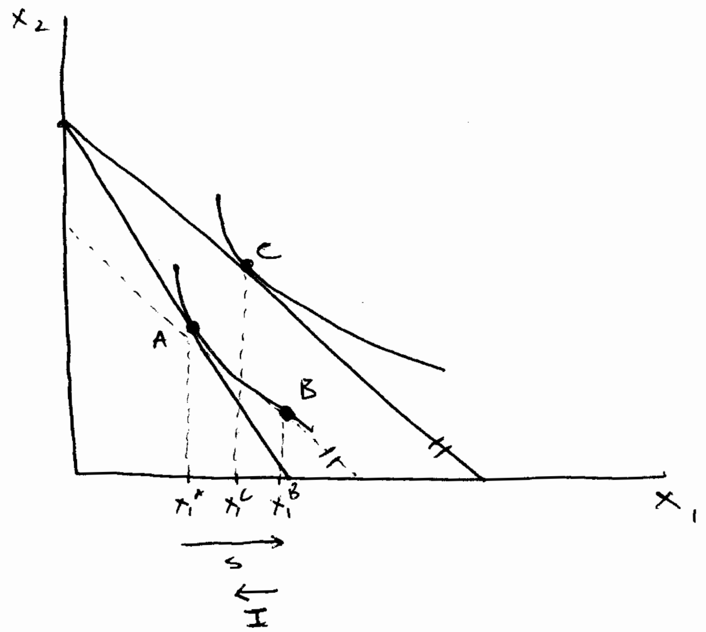
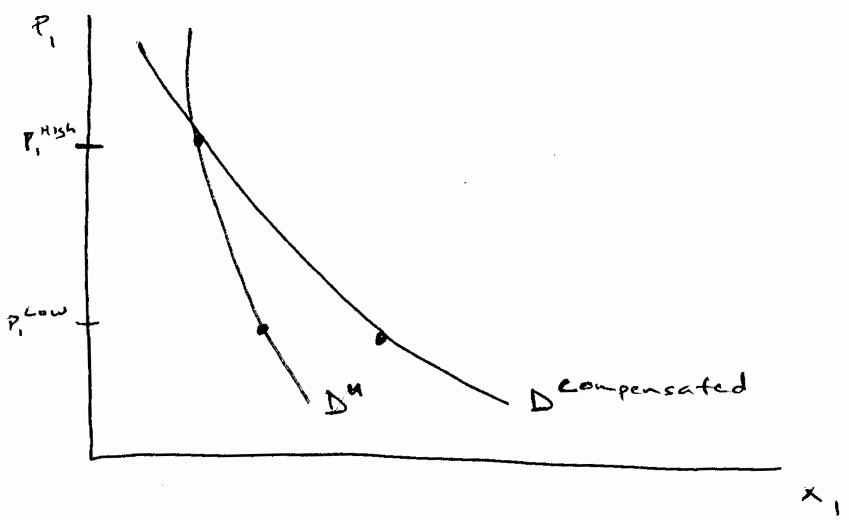
Practice Review One — Answer Key

I

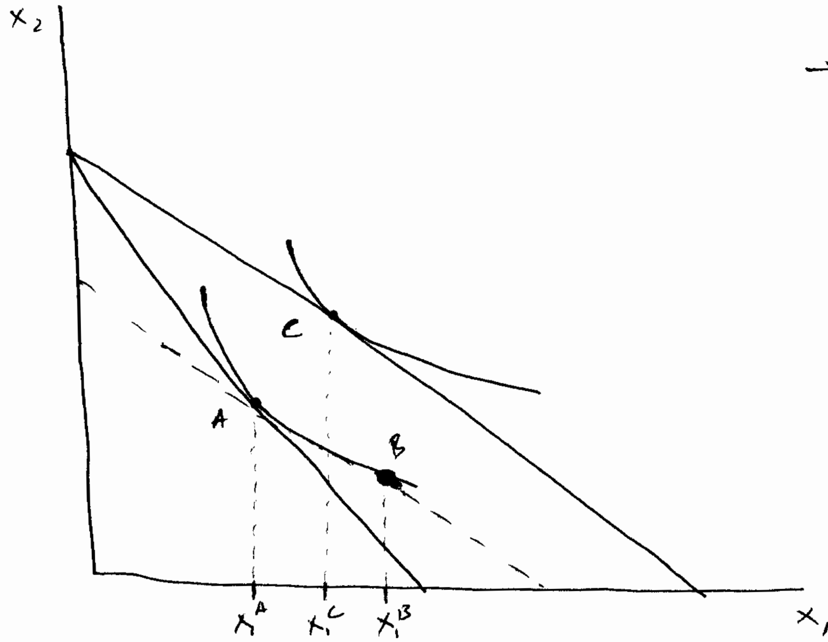
1





False

2



→ Counter Example:

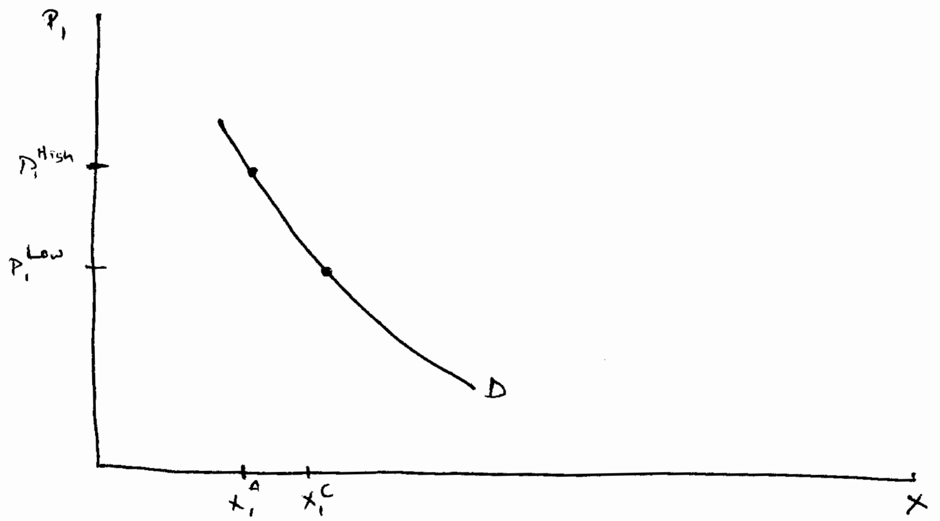
Price of Good 1 falls,

Good 1 is inferior.

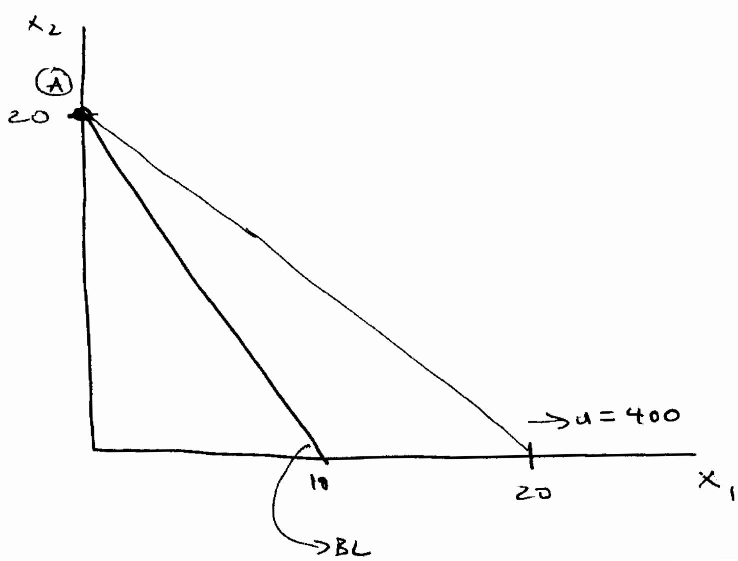
Consumption of good 1

goes up.

$\boxed{\text{False}}$



3



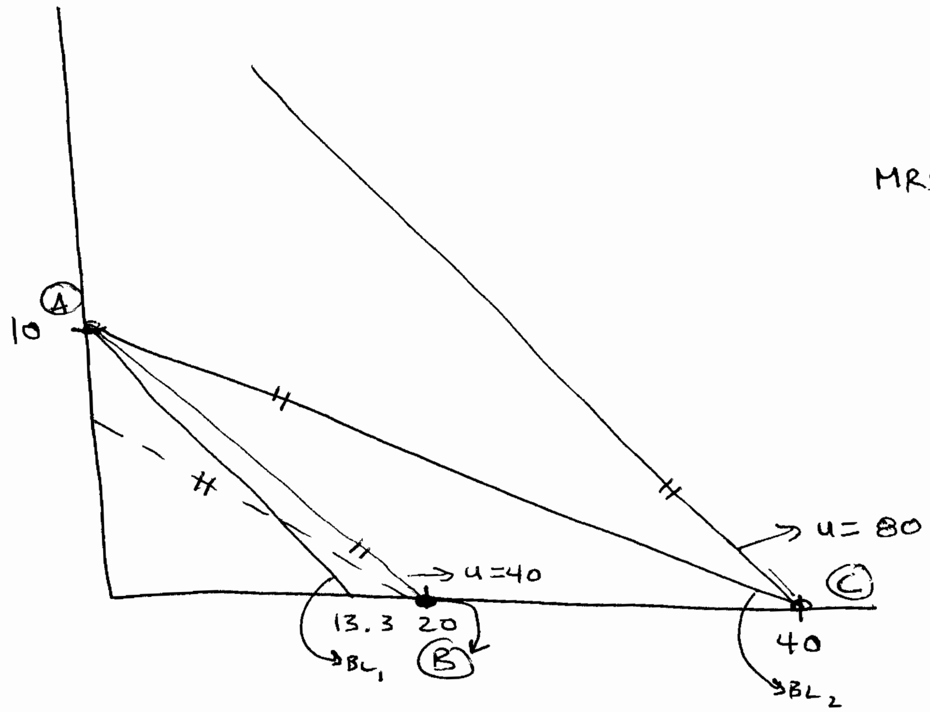
MRS = 1

A is utility maximizing bundle,

so

$\boxed{\text{False}}$

4



A to B ⇒ Substitution Effect

B to C ⇒ Income Effect

$\boxed{\text{False}}$

5

$$X_1 = \frac{1}{4} \cdot \frac{I}{P_1} \Rightarrow \text{demand for } X_1$$

$$\underbrace{X_1 \cdot P_1}_{\text{Expenditure on good 1}} = \frac{I}{4} \quad \therefore \quad \boxed{\text{True}}$$

II

1a) $\mathcal{L} = x_1^{1/2} + x_2^{1/2} - \lambda(P_1x_1 + P_2x_2 - I)$

$\mathcal{L}_{x_1} = \frac{1}{2}x_1^{-1/2} - \lambda P_1 = 0$

$\mathcal{L}_{x_2} = \frac{1}{2}x_2^{-1/2} - \lambda P_2 = 0$

$\mathcal{L}_{\lambda} = -(P_1x_1 + P_2x_2 - I) = 0$

$\frac{1}{2\sqrt{x_1}\,P_1} = \frac{1}{2\sqrt{x_2}\,P_2}$

① $\boxed{x_1P_1^2 = x_2P_2^2}$

② $\boxed{P_1x_1 + P_2x_2 = I}$

$P_1\left(x_2\frac{P_2^2}{P_1^2}\right) + P_2x_2 = I$

$x_2\left(\frac{P_2^2}{P_1} + P_2\right) = I$

$$\boxed{x_2 = \frac{I}{\left(\frac{P_2^2}{P_1} + P_2\right)}}$$

$P_1x_1 + P_2\left(x_1\frac{P_1^2}{P_2^2}\right) = I$

$$\boxed{x_1 = \frac{I}{\left(P_1 + \frac{P_1^2}{P_2}\right)}}$$

1c)

If $P_1 = P_2 = 1$

$x_1 = \frac{I}{2}$

$x_2 = \frac{I}{2}$

$$\mathcal{L} = p_1 x_1 + p_2 x_2 - \lambda\left(x_1^{1/2} + x_2^{1/2} - u\right)$$

$$\left.\begin{aligned} \mathcal{L}_{x_1} &= p_1 - \lambda \tfrac{1}{2} x_1^{-1/2} = 0 \\ \mathcal{L}_{x_2} &= p_2 - \lambda \tfrac{1}{2} x_2^{-1/2} = 0 \end{aligned}\right\} \Rightarrow p_1 x_1^{1/2} = p_2 x_2^{1/2}$$

① $\boxed{p_1^2 x_1 = p_2^2 x_2}$

$$\mathcal{L}_\lambda = -\left(x_1^{1/2} + x_2^{1/2} - u\right) = 0$$

↳ ② $\boxed{x_1^{1/2} + x_2^{1/2} = u}$

$$x_1^{1/2} + \left(\frac{p_1^2}{p_2^2} x_1\right)^{1/2} = u$$

$$x_1^{1/2}\left(1 + \frac{p_1}{p_2}\right) = u$$

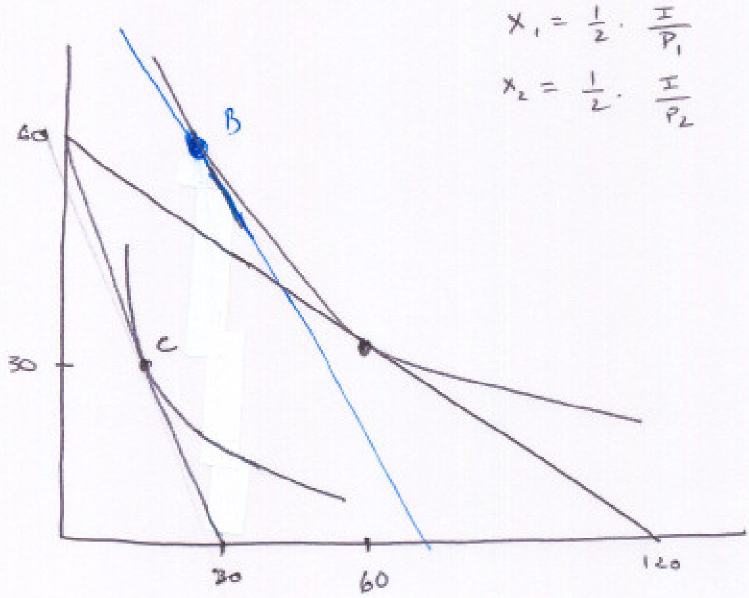
$$x_1^{1/2} = \left[\frac{p_2}{p_1 + p_2}\right] u$$

$$\boxed{x_1^c = \left[\frac{p_2}{p_1+p_2}\right]^2 u^2}$$

$$x_2^c = \frac{p_1^2}{[p_1 + p_2]^2} u^2$$

$$e(p_1, p_2, u) = p_1\left[\frac{p_2^2}{(p_1+p_2)^2} u^2\right] + p_2\left[\frac{p_1^2}{(p_1+p_2)^2} u^2\right]$$

2



$x_1 = \frac{1}{2} \cdot \frac{I}{p_1}$

$x_2 = \frac{1}{2} \cdot \frac{I}{p_2}$

A: $x_1 = 60$, $x_2 = 30$

B: $x_1 = 30$, $x_2 = 60$

C: $x_1 = 15$, $x_2 = 30$

B $\mathcal{L} = 2x_1 + x_2 - \lambda(x_1 x_2 - 1800)$

$\mathcal{L}_{x_1} = 2 - \lambda x_2 = 0$ $\qquad \frac{2}{x_2} = \lambda = \frac{1}{x_1}$

$\mathcal{L}_{x_2} = 1 - \lambda x_1 = 0$ $\qquad \boxed{2x_1 = x_2}$

$\mathcal{L}_\lambda = -(x_1 x_2 - 1800) = 0$

$$x_1 \, 2x_1 = 1800$$

$$x_1^2 = 900$$

$$x_1 = 30 \qquad x_2 = 60$$

A to B $\Rightarrow$ Substitution Effect

B to C $\Rightarrow$ Income Effect

4 a $\boxed{2x_1 = x_2}$ ② $\boxed{P_1 x_1 + P_2 x_2 = I}$

$$P_1 x_1 + P_2 \, 2x_1 = I$$

$$\boxed{x_1 = \frac{I}{P_1 + 2P_2}}$$

$$\frac{dx_1}{dP_1} = -\frac{I}{(P_1 + 2P_2)^2} = -\frac{100}{(1+2)^2} = -\frac{100}{9}$$

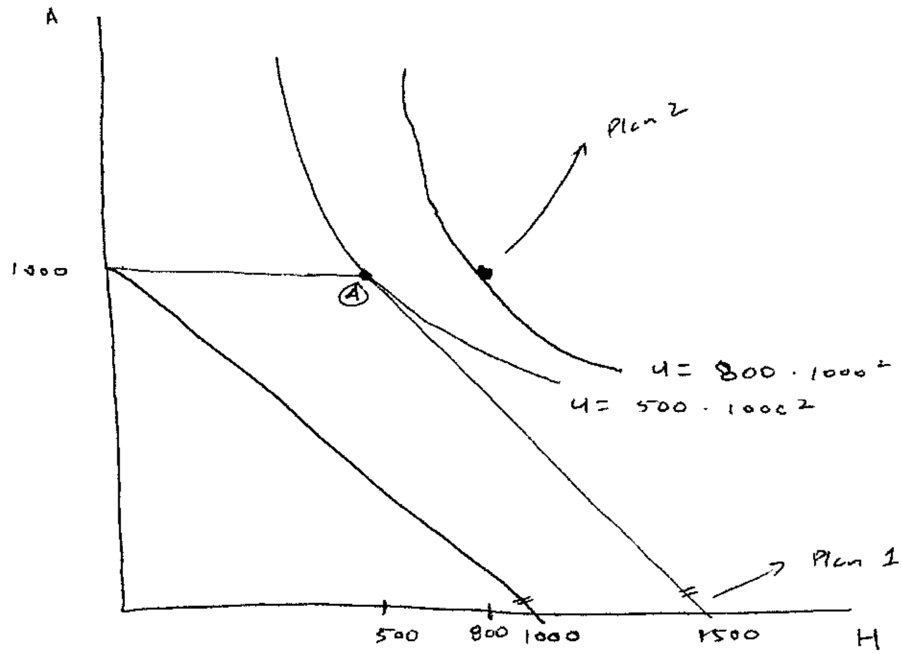
$$\frac{dx_1}{dI} = \frac{1}{(P_1 + 2P_2)} = \frac{1}{(1+2)} = \frac{1}{3}$$

$$x_1 = \frac{100}{(1+2)} = \frac{100}{3}$$

$$-\frac{100}{9} = \frac{dx_1^c}{dP_1} - \frac{100}{3} \cdot \left(\frac{1}{3}\right) \Rightarrow \frac{dx_1^c}{dP_1} = \phi$$

#3

a)



b) Plan 1

$H = \frac{1}{3} \cdot \frac{I}{P_H} \qquad A = \frac{2}{3} \cdot \frac{I}{P_A}$

$H^* = \frac{1}{3} \cdot \frac{1500}{1} = 500 \qquad A = \frac{2}{3} \cdot 1500/1 = 1000$

$U = 500 (1000)^2$

Plan 2 $\Rightarrow$ $U = 800((1000)^2$ $\boxed{\text{Choose Plan 2}}$

c) $\mathcal{L} = H + A - \lambda (HA^2 - 800 (1000)^2)$

$\left.\begin{array}{l} \mathcal{L}_H = 1 - \lambda A^2 = 0 \\ \mathcal{L}_A = 1 - \lambda 2HA = 0 \end{array}\right\}$ $\frac{1}{A^2} = \frac{1}{2HA}$

$\mathcal{L}_\lambda = -(HA^2 - 800(1000)^2) = 0$ $\boxed{2H = A}$

d) Price would fall; demand decreased.

$H(2H)^2 = 800 \cdot 1000^2$

$H^* = 584.8 \qquad A^* = 1169.6$

Cost $= 1754.40 \Rightarrow$ Give each family $1754.40 - 1000 = 754.40$

total cost $= 754.40 \times 50 = 37,720$