Eco 202

Smith

Problem Set 2 – Answer Key

$\boxed{\text{I}}$

$\boxed{1}$ $u(x, y) = ax + by$ perfect substitutes

$$MRS = \frac{a}{b} = 1 \quad \underline{\text{if}} \quad a = b$$

$$u(x, y) = x^2 + 2xy + y^2$$

$$MRS = \frac{2x + 2y}{2x + 2y} = 1$$

<u>True</u>, it does represent perfect substitutes preferences.

$\boxed{2}$ True.

If a consumer has a utility function given by the expression

$$u = f(x, y)$$

then any utility function such that

$$\hat{u} = \alpha \cdot f(x, y)$$

where $\alpha > 0$ will order bundles in such a way that the

$$MRS_u = MRS_{\hat{u}} \quad \text{since}$$

$$\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = \frac{\alpha \cdot \frac{\partial f}{\partial x}}{\alpha \cdot \frac{\partial f}{\partial y}}.$$

II

3

a) $u(x, y) = x^a y^b$

$$MRS = \frac{a x^{a-1} y^b}{b x^a y^{b-1}} = \frac{a y}{b x}$$

$u(x, y) = 10 x^a y^b + 2000$ ✓

$$MRS = \frac{10 a x^{a-1} y^b}{10 b x^a y^{b-1}} = \frac{a y}{b x}$$

b) $u(x, y) = x + y$

$$MRS = \frac{1}{1} = 1$$

$u(x, y) = (x + y)^3$ ✓

$$MRS = \frac{3(x+y)^2 \cdot 1}{3(x+y)^2 \cdot 1} = 1$$

c) $u(x, y) = x^3 y^3$

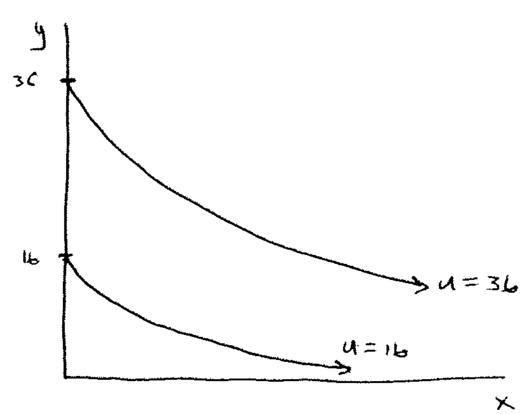
$$MRS = \frac{3x^2 y^3}{3x^3 y^2} = \frac{y}{x}$$

$u(x, y) = a^3 x^{10} y^{10}$ ✓

$$MRS = \frac{10 a^3 x^9 y^{10}}{10 a^3 x^{10} y^9} = \frac{y}{x}$$

4

a)



Let $u = 16$

$16 = xy + y$

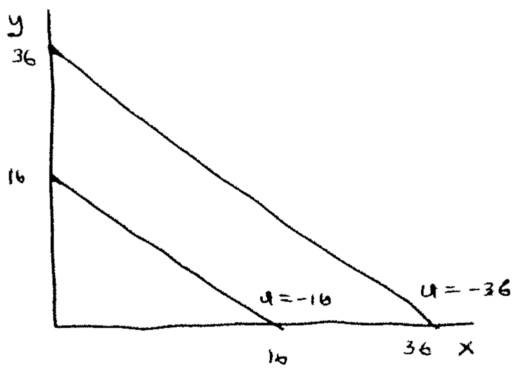
$$y = \frac{16}{x+1}$$

4a) → Monotonicity? Look for one example where it isn't true. If it fails for even one example, then the property fails to hold. There are no examples of "more is better" failing to provide greater utility in this case, so monotonicity holds.

→ Consumer likes moderate amounts of both goods, but must have some <u>y</u> or her utility fall to zero.

→ $MRS = \frac{y}{x+1}$

b)



$u=16$, $u=36$

<u>Not</u> possible to have $u>0$ if $x \geq 0$ and $y \geq 0$.

So, graph $u=-16$ $u=-36$.

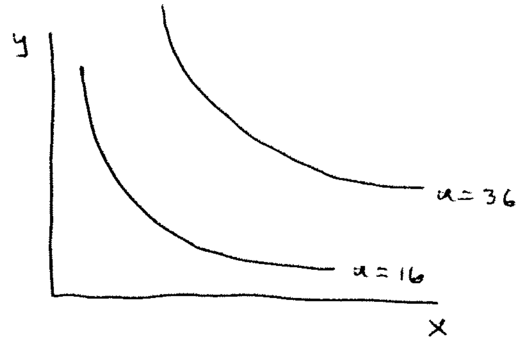
The $u=-16$ indifference curve is <u>better</u> than the $u=-36$ indifference curve.

→ $MRS = \frac{-1}{-1} = 1$

→ <u>Not</u> monotonic; more is always <u>worse</u>.

→ These are economic "bads" that are perfect substitutes for one another.

4c)



$u(x,y) = x^2 y^2$

$u=16 \quad 16 = x^2 y^2$

$y^2 = \frac{16}{x^2}$

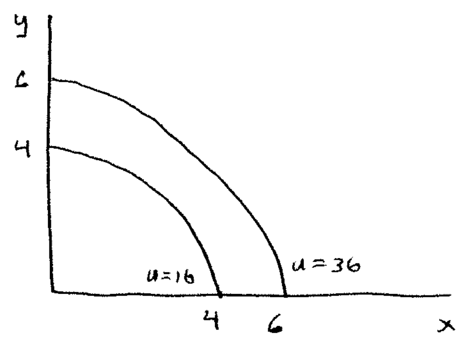
$y = \frac{4}{x}$

→ $MRS = \frac{2xy^2}{2x^2y} = \frac{y}{x}$

→ Does satisfy monotonicity.

→ Consumers prefer moderate amounts of each good, but must have some of each or else will end up with zero utility!

4d)



$u(x,y) = x^2 + y^2$

$u=16 \quad y^2 = 16 - x^2$

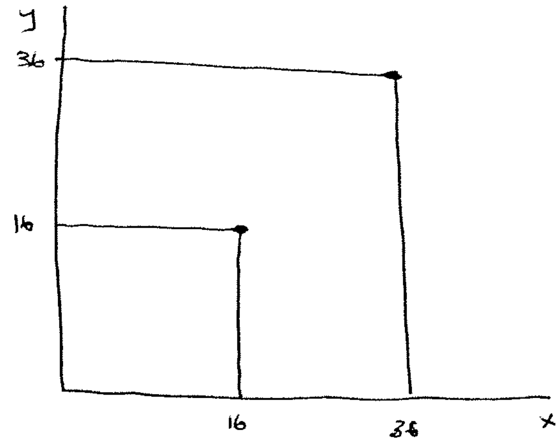
$y = \sqrt{16 - x^2}$

→ $MRS = \frac{2x}{2y} = \frac{x}{y}$

→ Does satisfy monotonicity.

→ "Extreme" Preferences — A consumer will typically choose bundles with one good or the other. (You'll see why in the section on consumer choice!)

4e) $u(x, y) = \text{Max}\{x, y\}$



$u = 16$ $\quad 16 = \text{Max}\{x, y\}$

x	y
16	0
0	16
16	16
8	16
16	8

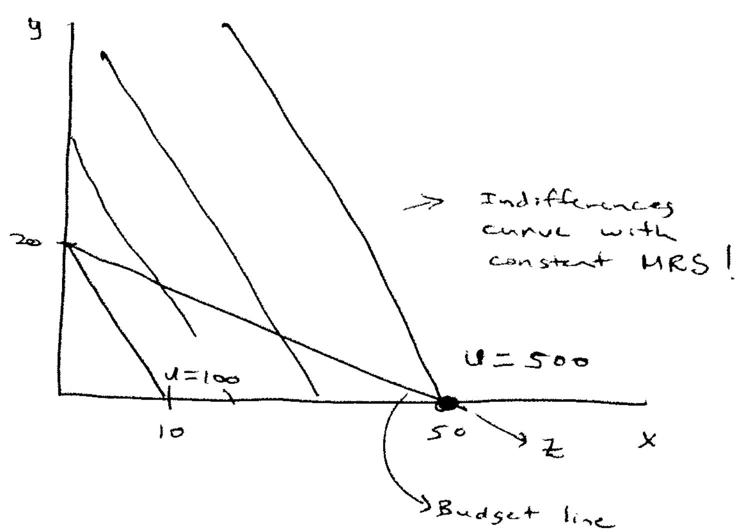
→ MRS = 0, undefined, or ∞

→ Does not satisfy monotonicity. If you are at $x = 16$, $y = 36$ and move to $x = 17$, $y = 36$, then your utility does not change.

→ "Extreme" Preferences — consumers will, once again, typically choose bundles with one good or the other.

$\boxed{\text{III}}$

5a) $u(x, y) = 10x + 5y$ $\quad$ MRS = 2



$2x + 5y = 100$

Bundle Z

$x = 50$, $y = 0$

maximizes utility.

5b) $u(x,y) = x^{1/4} y^{3/4}$

$$\mathcal{L} = x^{1/4} y^{3/4} - \lambda(2x + 5y - 100)$$

$$\mathcal{L}_x = \tfrac{1}{4} x^{-3/4} y^{3/4} - 2\lambda = 0$$

$$\mathcal{L}_y = \tfrac{3}{4} x^{1/4} y^{-1/4} - 5\lambda = 0$$

$$\mathcal{L}_\lambda = -(2x + 5y - 100) = 0$$

$$\frac{\tfrac{1}{4} x^{-3/4} y^{3/4}}{2} = \lambda = \frac{\tfrac{3}{4} x^{1/4} y^{-1/4}}{5}$$

$\downarrow$

$$\frac{\tfrac{1}{4} x^{-3/4} y^{3/4}}{\tfrac{3}{4} x^{1/4} y^{-1/4}} = \frac{2}{5}$$

① $\boxed{\frac{y}{3x} = \frac{2}{5}}$

② $\boxed{2x + 5y = 100}$

① → ②

$y = \frac{6}{5} x$

$\downarrow$

$2x + 5\left(\frac{6}{5} x\right) = 100$

$8x = 100$

$\boxed{x = 12.5 \quad y = 15}$

c) $u(x,y) = \text{Min}\{x, 2y\}$

$x = 2y \qquad y = x/2$

$2x + 5y = 100$

$2x + 5\left(\frac{x}{2}\right) = 100$

$\frac{9x}{2} = 100$

$\boxed{x^* = \frac{200}{9} \qquad y^* = \frac{100}{9}}$

d) $u(x,y) = x^{1/2} + y$

$\mathcal{L} = x^{1/2} + y - \lambda(2x + 5y - 100)$

$\left.\begin{array}{l} \mathcal{L}_x = \frac{1}{2}x^{-1/2} - 2\lambda = 0 \\ \mathcal{L}_y = 1 - 5\lambda = 0 \end{array}\right\} \quad \frac{1}{4\sqrt{x}} = \frac{1}{5}$

$\mathcal{L}_\lambda = -(2x + 5y - 100) = 0$

$\sqrt{x} = \frac{5}{4} \qquad \boxed{x^* = \frac{25}{16}}$

$2x + 5y = 100$

$2\left(\frac{25}{16}\right) + 5y = 100$

$5y = 100 - \frac{50}{16}$

$\boxed{\begin{array}{l} y^* = 19.375 \\ x^* = 1.5625 \end{array}}$

e) $u(x,y) = xy + x$

$$\mathcal{L} = xy + x - \lambda(2x + 5y - 100)$$

$$\mathcal{L}_x = y + 1 - 2\lambda = 0$$

$$\mathcal{L}_y = x - 5\lambda = 0$$

$$\mathcal{L}_\lambda = -(2x + 5y - 100) = 0$$

② $\boxed{2x + 5y = 100}$

$$\frac{y+1}{2} = \frac{x}{5}$$

$$5y + 5 = 2x$$

① $\boxed{x = \frac{5y+5}{2}}$

① → ② $\quad 5y + 5 + 5y = 100$

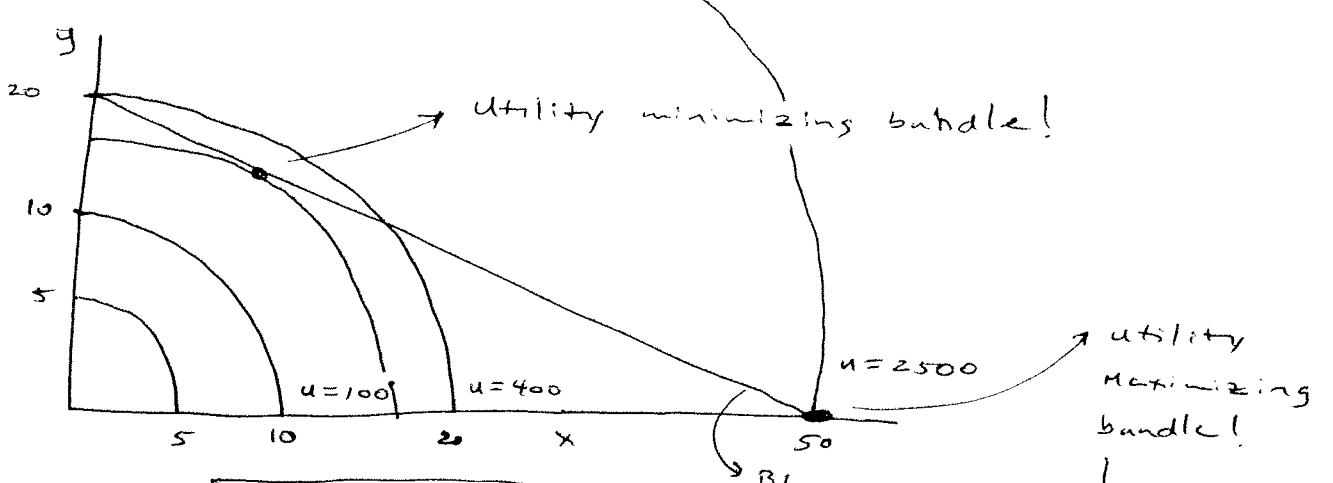
$$10y = 95$$

$$\boxed{\begin{aligned} y^* &= 9.5 \\ x^* &= 26.25 \end{aligned}}$$

f) $u(x,y) = x - y \Rightarrow$ y is an economic bad → spend no money on this good.

$$\boxed{\begin{aligned} x^* &= 50 \\ y^* &= 0 \end{aligned}}$$

g) $u(x,y) = x^2 + y^2$



$\boxed{x^* = 50 \quad y^* = 0}$