Problem Set 3 – Key

I

1 $\mathcal{L} = x^3 y - \lambda(P_x \cdot x + P_y \cdot y - I)$

$\mathcal{L}_x = 3x^2 y - \lambda P_x = 0$

$\mathcal{L}_y = x^3 - \lambda P_y = 0$

$\mathcal{L}_\lambda = -(P_x \cdot x + P_y \cdot y - I) = 0$ ②

$$P_x \cdot x + P_y \cdot y = I$$

$\frac{3x^2 y}{P_x} = \frac{x^3}{P_y}$ $\quad \frac{3y}{x} = \frac{P_x}{P_y}$

① $y = \frac{P_x}{P_y} \cdot \frac{x}{3}$ $\qquad$ ① $x = \frac{P_y}{P_x} 3y$

Plug ① into ②

$P_x\left(\frac{P_y}{P_x} \cdot 3y\right) + P_y \cdot y = I$

$4 P_y \cdot y = I$

$$y = \frac{1}{4} \cdot \frac{I}{P_y}$$

$P_x \cdot x + P_y\left(\frac{P_x}{P_y} \cdot \frac{x}{3}\right) = I$

$4 P_x \cdot x = 3I$

$$x = \frac{3}{4} \cdot \frac{I}{P_x}$$

2

1) Line that connects the corners:

$8x = 4y$

$x = 1/2\, y$ ① $\quad y = 2x$

2) Budget line: ② $P_x \cdot x + P_y \cdot y = I$

3) Plug ① → ②

$P_x \cdot \frac{1}{2} y + P_y \cdot y = I$

$$y = \frac{I}{[P_x/2 + P_y]}$$

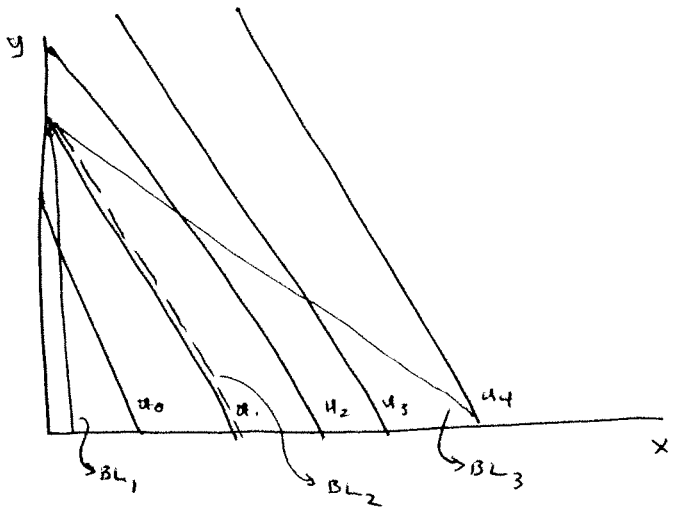
$P_x \cdot x + P_y \cdot 2x = I$

$$x = \frac{I}{[P_x + 2P_y]}$$

(3)

$u = 3x + y \quad \Rightarrow \quad MRS = \frac{3}{1}$

Solve using graphical analysis:



Case 1: (BL_1)

If $\frac{P_x}{P_y} > MRS$

$x^* = 0$

$y^* = \frac{I}{P_y}$

Case 2: (BL_2)

If $\frac{P_x}{P_y} = MRS$

$x^* = [0, I/P_x]$

$y^* = [0, I/P_y]$

Case 3: BL_3

If $P_x/P_y < MRS$

$x^* = \frac{I}{P_x} \qquad y^* = 0$

So, each demand has three "parts":

$$x = \begin{cases} 0 & \text{if } P_x/P_y > MRS \\ [0, I/P_x] & \text{if } P_x/P_y = MRS \\ I/P_x & \text{if } P_x/P_y < MRS \end{cases} \qquad y = \begin{cases} 0 & \text{if } P_x/P_y < MRS \\ [0, I/P_y] & \text{if } P_x/P_y = MRS \\ I/P_y & \text{if } P_x/P_y > MRS \end{cases}$$

[4] $\mathcal{L} = \ln x + y - \lambda(P_x \cdot x + P_y \cdot y - I)$

$\mathcal{L}_x = 1/x - \lambda P_x = 0$

$\mathcal{L}_y = 1 - \lambda P_y = 0$

$\mathcal{L}_\lambda = -(P_x \cdot x + P_y \cdot y - I) = 0$

$\Big\} \quad \frac{1}{x \cdot P_x} = \frac{1}{P_y}$

① $\boxed{x = \frac{P_y}{P_x}}$

↳ ② $\boxed{P_x \cdot x + P_y \cdot y = I}$

Note equation one is a demand function!

$\boxed{x = \frac{P_y}{P_x}}$

① → ② $P_x\left(\frac{P_y}{P_x}\right) + P_y \cdot y = I$

$\boxed{y = \frac{I}{P_y} - 1}$

[5] $\mathcal{L} = x \cdot y + x - \lambda(P_x \cdot x + P_y \cdot y - I)$

Note: ★ Not Cobb Douglas! ★

$\mathcal{L}_x = y + 1 - \lambda P_x = 0$

$\mathcal{L}_y = x - \lambda P_y = 0$

$\mathcal{L}_\lambda = -(P_x \cdot x + P_y \cdot y - I) = 0$

↳ ② $\boxed{P_x \cdot x + P_y \cdot y = I}$

① $\boxed{\frac{y+1}{P_x} = \frac{x}{P_y}}$

$x = \frac{P_y}{P_x}(y+1)$ $\qquad y = x\frac{P_x}{P_y} - 1$

① → ②

$P_x\left(\frac{P_y}{P_x}(y+1)\right) + P_y \cdot y = I$

$P_y \cdot y + P_y + P_y \cdot y = I$

$2P_y y = I - P_y$

$\boxed{y = \frac{1}{2} \cdot \frac{I}{P_y} - \frac{1}{2}}$

$P_x \cdot x + P_y\left(\frac{P_x}{P_y}x - 1\right) = I$

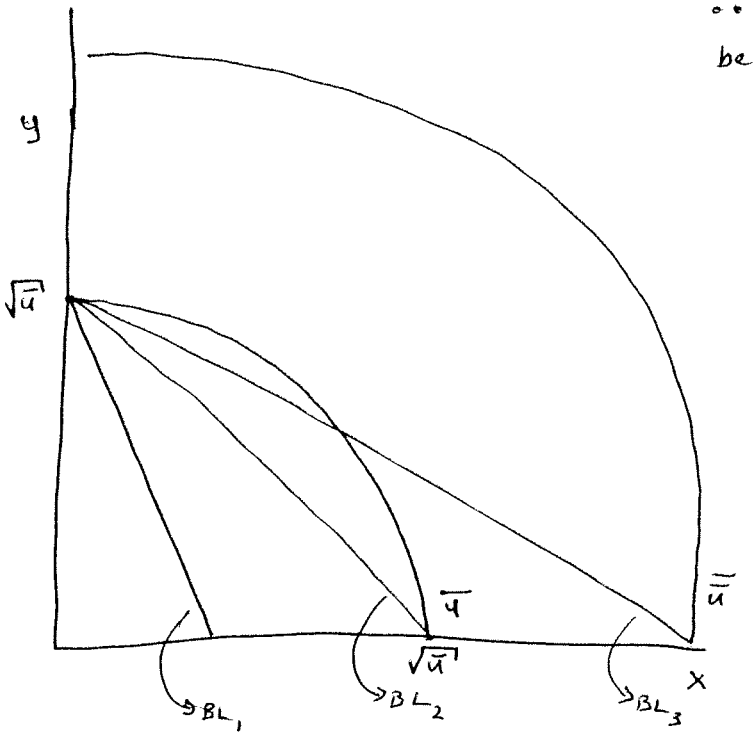
$2P_x \cdot x - P_y = I$

$\boxed{x = \frac{1}{2}\frac{I}{P_x} + \frac{P_y}{2P_x}}$

6 $u = x^2 + y^2 \rightarrow MRS = \frac{2x}{2y} = \frac{x}{y}$

So, MRS is not diminishing

∴ preferences are not well behaved.



Use graphical analysis:

Case 1: (BL_1)

If $\frac{P_x}{P_y} > 1$,

then

$x = 0$

$y = \frac{I}{P_y}$

Case 2: (BL_2)

If $P_x/P_y = 1$,

then

$x = 0 \quad y = I/P_y$

or

$x = I/P_x \quad y = 0$

Case 3: (BL_3)

If $\frac{P_x}{P_y} < 1$,

then

$x = I/P_x \quad y = 0$

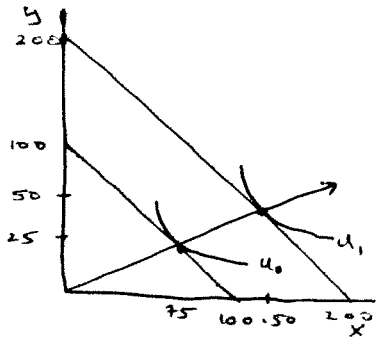
So, the demand functions:

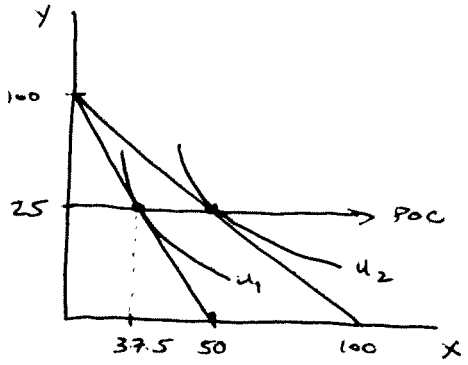
$$x = \begin{cases} 0 & \text{if } \frac{P_x}{P_y} \geq 1 \\ \frac{I}{P_x} & \text{if } \frac{P_x}{P_y} \leq 1 \end{cases}$$

$$y = \begin{cases} 0 & \text{if } \frac{P_x}{P_y} \leq 1 \\ \frac{I}{P_y} & \text{if } \frac{P_x}{P_y} \geq 1 \end{cases}$$

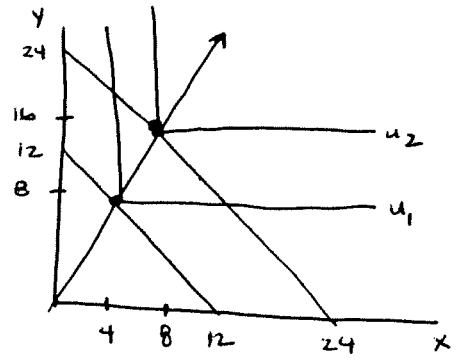
II

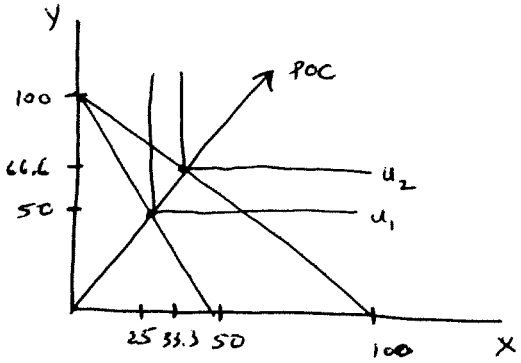
1.


y
200
100
50
25
u_1
u_0
75
100
150
200
x

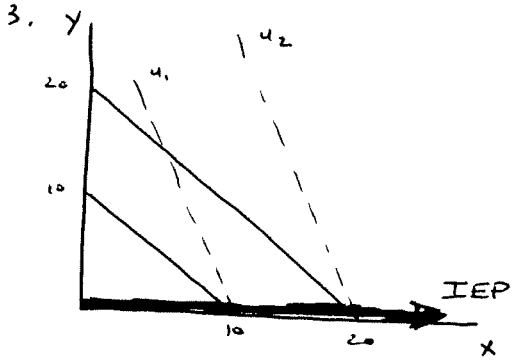

y
100
25
PoC
u_2
u_1
37.5
50
100
x

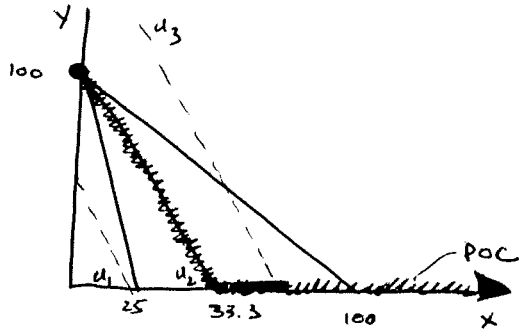
2.


y
24
16
12
8
u_2
u_1
4
8
12
24
x

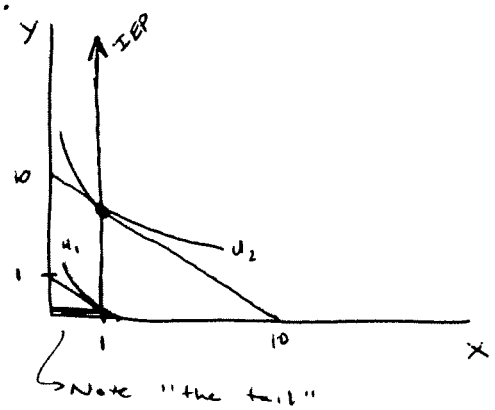

y
100
66.6
50
PoC
u_2
u_1
25
33.3
50
100
x

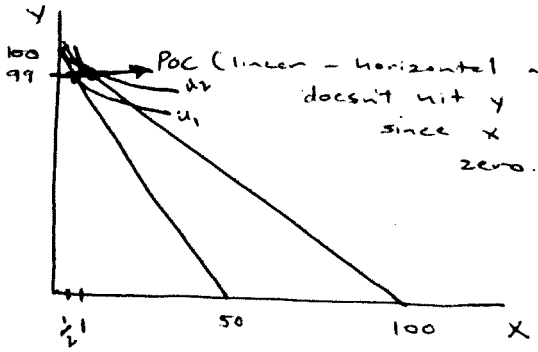
3.


y
u_2
u_1
20
10
IEP
10
20
x


y
u_3
100
u_1
u_2
PoC
25
33.3
100
x

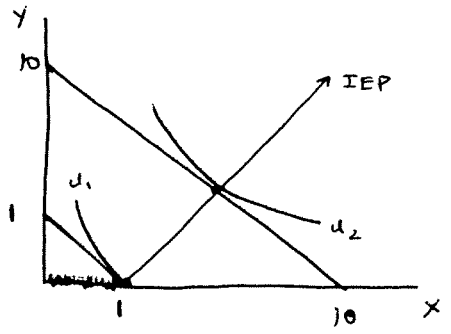
4.

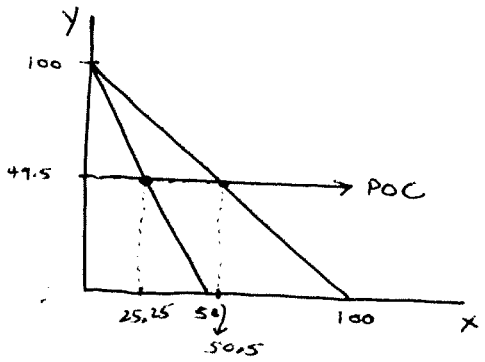

y
IEP
10
1
u_1
u_2
1
10
x
Note "the tail"


y
100
99
PoC (linear – horizontal at y = 99, doesn't hit y axis since x can't be zero.)
u_2
u_1
½
1
50
100
x

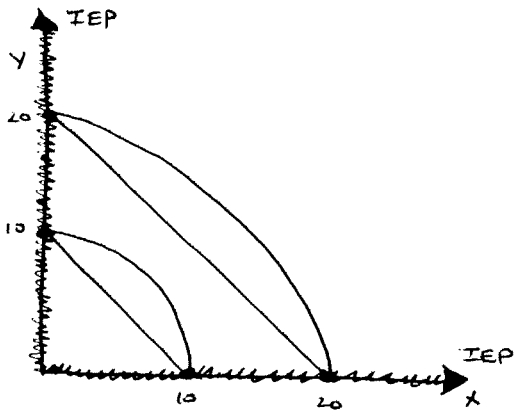
II

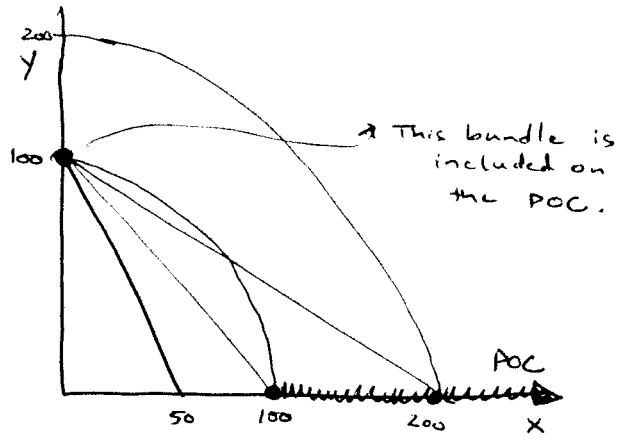
(5)





(6)



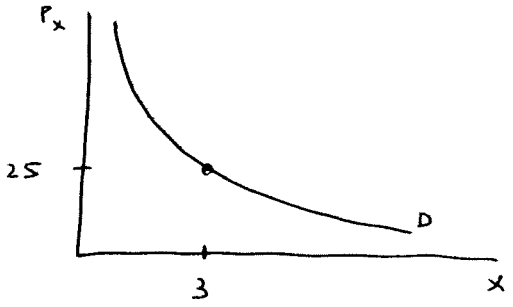


III

(1)

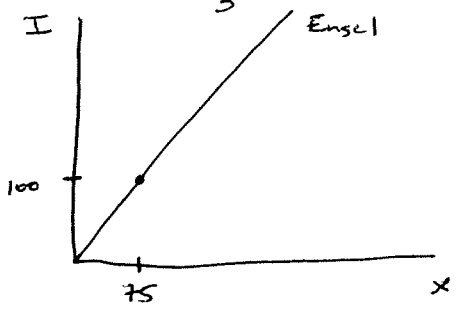
$$X = \frac{3}{4} \cdot \frac{100}{P_x}$$

$$P_x = \frac{75}{X}$$



$$X = \frac{3}{4} \cdot \frac{I}{1}$$

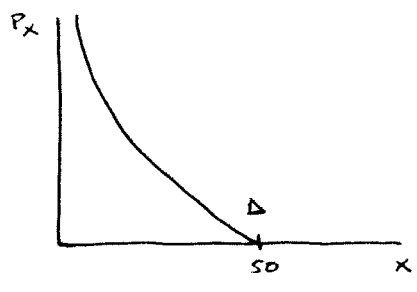
$$I = \frac{4}{3}X$$



(2)

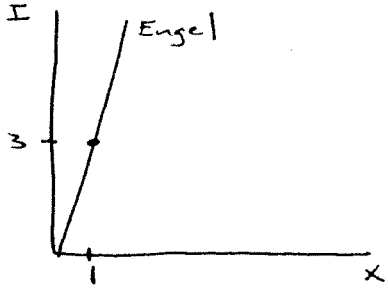
$$X = \frac{100}{P_x + 2}$$

$$P_x = \frac{100}{x} - 2$$

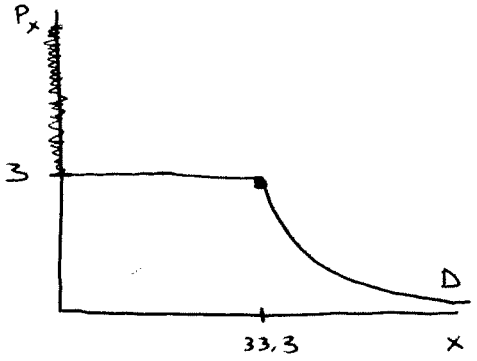


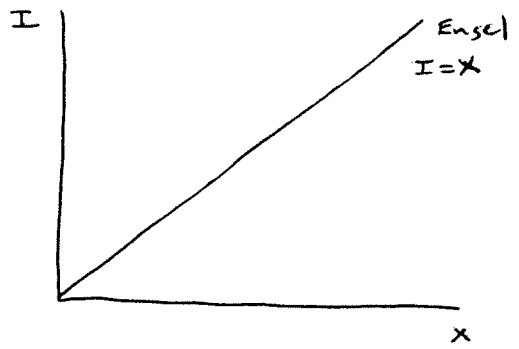
$$X = \frac{I}{1+2}$$

$$I = 3X$$



③

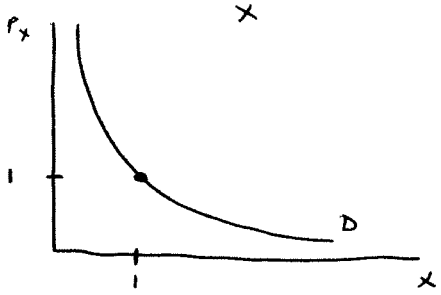


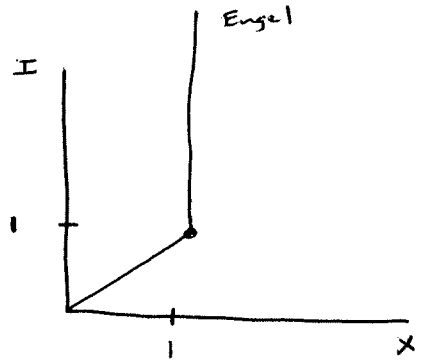


④

$$X = \frac{1}{P_x}$$

$$P_x = \frac{1}{X}$$



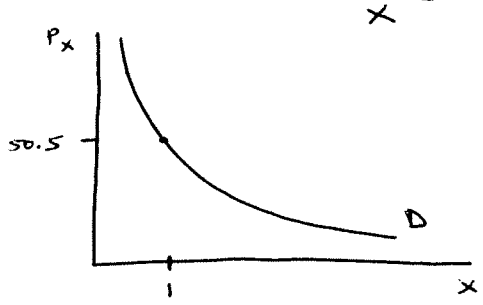


⑤

$$X = \frac{100}{2P_x} + \frac{1}{2P_x}$$

$$X = \frac{101}{2P_x}$$

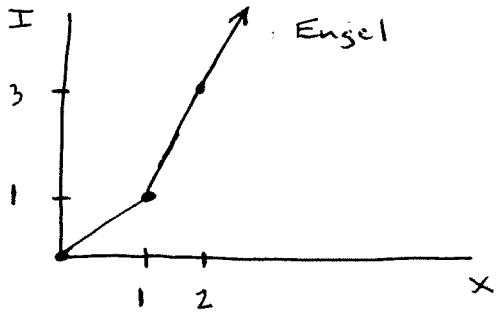
$$P_x = \frac{50.5}{X}$$



$$X = \frac{I}{2} + \frac{1}{2}$$

$$X = \frac{I+1}{2}$$

$$I = 2X - 1$$



⑥ Can be either $X = 0$ or $X = I/P_x$ because $P_y = 1$ and P_x is a variable.