Eco 202
Smith

Problem Set # 4 – Answer Key

I

① $f(\bar{K}, \bar{L}) = \bar{K}^2\bar{L}^2 = \bar{Q}$

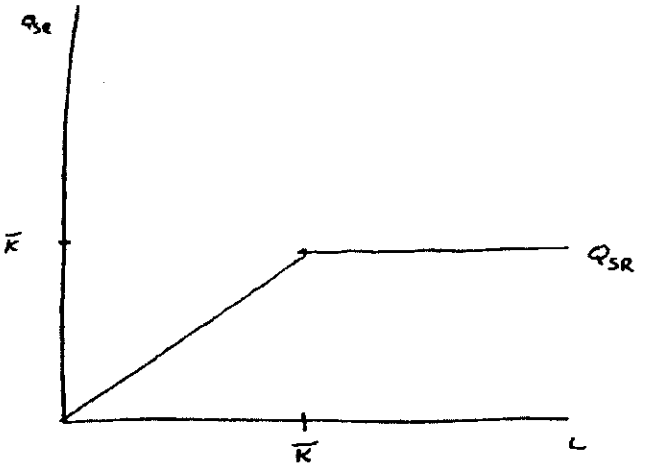
IRS

$f(t\bar{K}, t\bar{L}) = (t\bar{K})^2(t\bar{L})^2 = t^4(\bar{K}^2)(\bar{L}^2) = t^4\bar{Q} > t\bar{Q}$

$MP_L = \frac{\partial F}{\partial L} = 2K^2L \qquad \frac{\partial MP_L}{\partial L} = 2K^2 > 0 \quad \text{if } K \neq 0$

MP_L is not diminishing.

② $Q_{SR} = \text{Min}\{\bar{K}, L\}$



MP_L is the slope of Q_{SR}.

→ MP_L is constant.

$\bar{Q} = f(\bar{K}, \bar{L}) = \text{Min}\{\bar{L}, \bar{K}\}$

$f(t\bar{K}, t\bar{L}) = \text{Min}\{t\bar{L}, t\bar{K}\} = t\bar{Q}$

CRS

(3) $\tilde{Q} = f(\tilde{K}, \tilde{L}) = 4\tilde{K} + 2\tilde{L}$

$f(t\tilde{K}, t\tilde{L}) = 4(t\tilde{K}) + 2(t\tilde{L})$

$= t\left[4\tilde{K} + 2\tilde{L}\right]$

$= t \cdot Q$

↓

CRS

$MP_L = 2$ $\qquad$ $MP_K = 4$

$\frac{\partial MP_L}{\partial L} = 0$ $\qquad$ $\frac{\partial MP_K}{\partial K} = 0$

↳ Not diminishing; MP_L is constant.

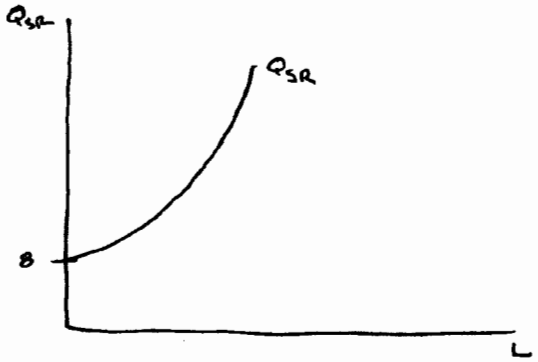
(4) $\tilde{Q} = f(\tilde{K}, \tilde{L}) = \frac{\tilde{K}^2}{2} + \frac{\tilde{L}^2}{2}$

$f(t\tilde{K}, t\tilde{L}) = \frac{(t\tilde{K})^2}{2} + \frac{(t\tilde{L})^2}{2} = t^2\left[\frac{\tilde{K}^2}{2} + \frac{\tilde{L}^2}{2}\right]$

$= t^2 \cdot \tilde{Q}$

↳ IRS

$MP_L = \frac{\partial f}{\partial L} = L$ $\qquad$ $\frac{\partial MP_L}{\partial L} = 1 > 0$ $\Rightarrow$ Not diminishing.



$\bar{K} = 4 \Rightarrow Q_{SR} = 8 + \frac{L^2}{2}$

(5) $\tilde{Q} = f(\tilde{K}, \tilde{L}) = \tilde{K}^{3/4} \tilde{L}^{3/4}$

$f(t\tilde{K}, t\tilde{L}) = (t\tilde{K})^{3/4} (t\tilde{L})^{3/4} = t^{6/4} \tilde{L}^{3/4} \tilde{K}^{3/4}$

$= t^{6/4} \tilde{Q} > t\tilde{Q}$

↳ **IRS**

$MP_L = \frac{3}{4} K^{3/4} L^{-1/4}$ $\qquad \frac{\downarrow MP_L}{\downarrow L} = -\frac{3}{16} K^{3/4} L^{-5/4} < 0$

↳ Diminishing MP_L !

(6) If $K = L = 1$

$Q = 1 \cdot 1 = 1$

$Q = 1^2 \cdot 1^2 = 1$

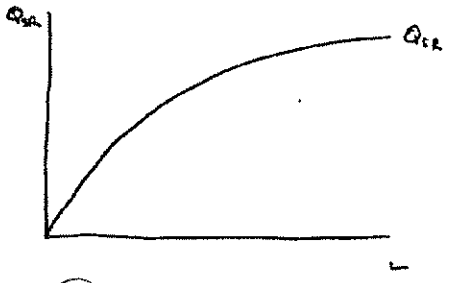
$K = L = 2$

$Q = 2 \cdot 2 = 4$

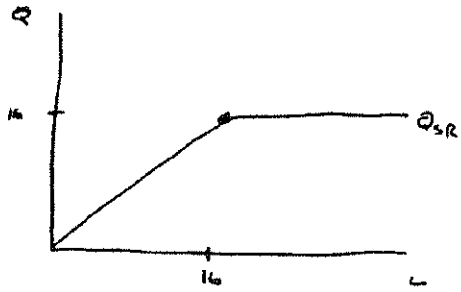
$Q = 2^2 \cdot 2^2 = 16$

} → Different output levels from the same input level.

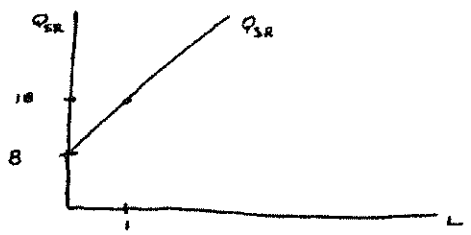
(8) $Q_{SR} = 2 \cdot L^{1/2}$



(9) $Q_{SR} = \text{Min}\{16, L\}$



(10)



$Q_{SR} = 8 + 10 \cdot L$

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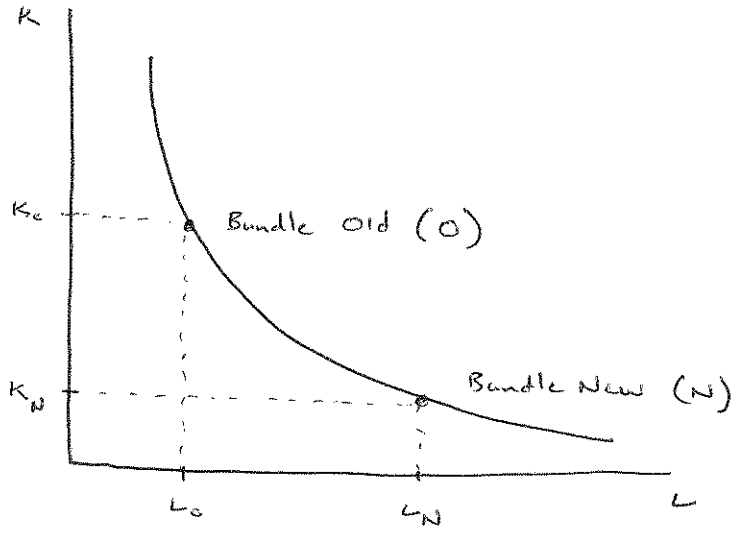
$$Q(K,L) = A \cdot K^{\alpha} L^{\beta}$$

$$MP_L = A\beta \cdot K^{\alpha} L^{\beta-1}$$

$$MP_K = A\alpha \cdot K^{\alpha-1} L^{\beta}$$



$$MRTS = \frac{MP_L}{MP_K} = \frac{A\beta K^{\alpha} L^{\beta-1}}{A\alpha K^{\alpha-1} L^{\beta}} = \frac{\beta K}{\alpha L}$$

So,

$$\sigma = \frac{\frac{K_N}{L_N} - \frac{K_0}{L_0}}{\frac{K_0}{L_0}} \Rightarrow \text{Multiply by } \frac{\beta}{\alpha} \Rightarrow \frac{\frac{\beta K_N}{\alpha L_N} - \frac{\beta K_0}{\alpha L_0}}{\frac{\beta K_0}{\alpha L_0}}$$

$$\frac{\frac{MRTS^N - MRTS^0}{MRTS^0}}{\frac{\frac{\beta K_N}{\alpha L_N} - \frac{\beta K_0}{\alpha L_0}}{\frac{\beta K_0}{\alpha L_0}}} \qquad \frac{\frac{\beta K_N}{\alpha L_N} - \frac{\beta K_0}{\alpha L_0}}{\frac{\beta K_D}{\alpha L_0}}$$

thus $\boxed{\sigma = 1}$