Answer Key — Problem Set # 5

1, 6, 11, 16

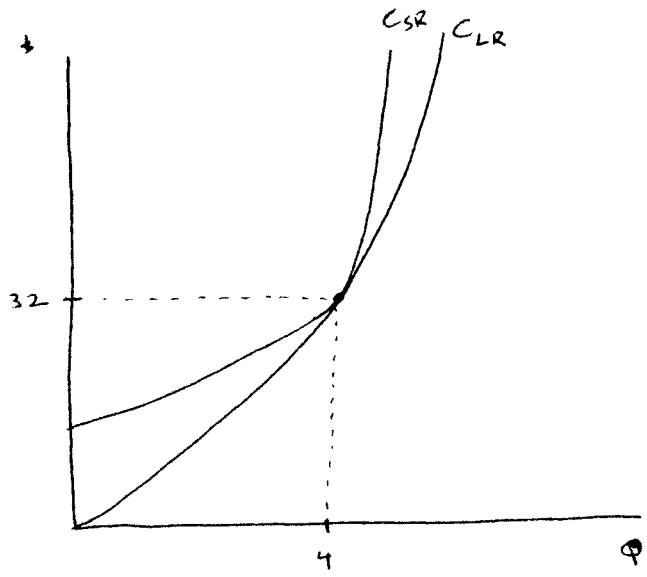
$Q = K^{1/4} L^{1/4}$

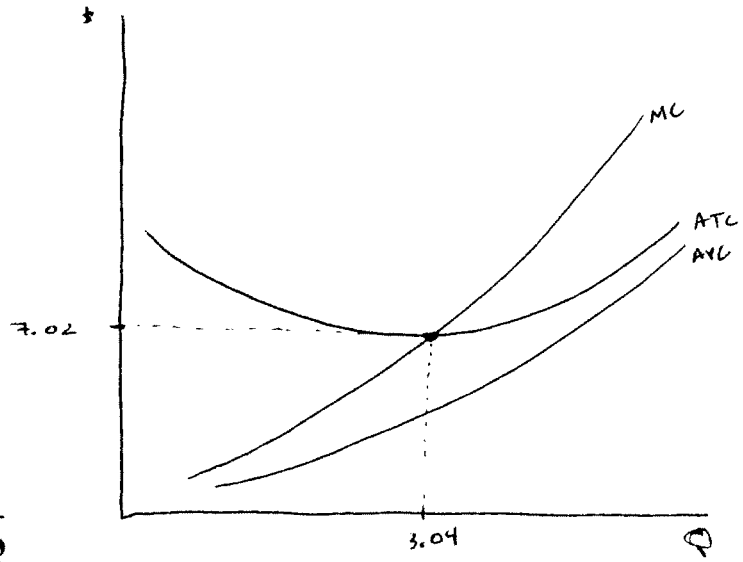
$L^* = w^{-1/2} r^{1/2} Q^2$

$C_{LR}(Q, w, r) = 2 w^{1/2} r^{1/2} Q^2$

$K^* = w^{1/2} r^{-1/2} Q^2$

$C_{SR}(Q, w, r) = w \cdot \frac{Q^4}{\bar{K}} + r \cdot \bar{K}$





$C_{LR} = 2Q^2$

$C_{SR} = \frac{Q^4}{16} + 16$

$MC = \frac{Q^3}{4}$

$AVC = \frac{Q^3}{16}$

$ATC = \frac{Q^3}{16} + \frac{16}{Q}$

i) DRS Technology — As inputs double, costs double; output does <u>not</u> double.

ii) After $Q = 3.04$ AVC dominates AFC and ATC begins to rise.

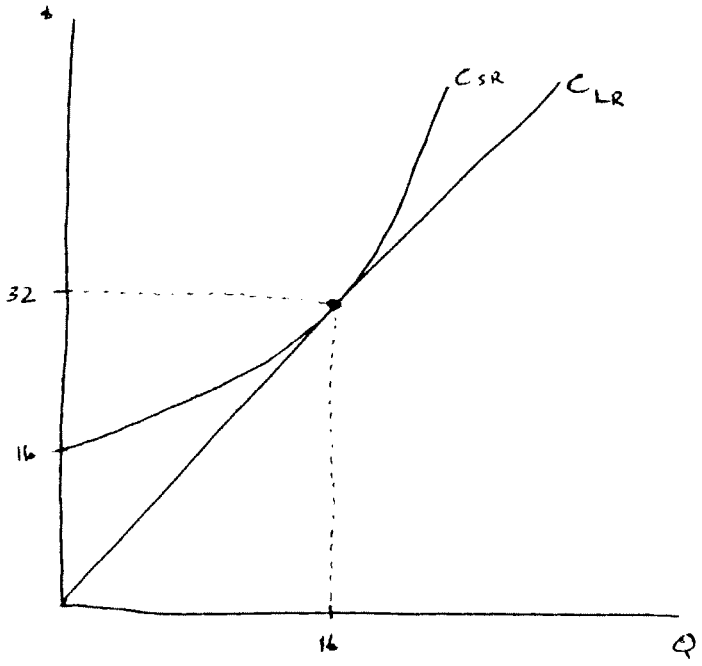
2, 7, 12, 17

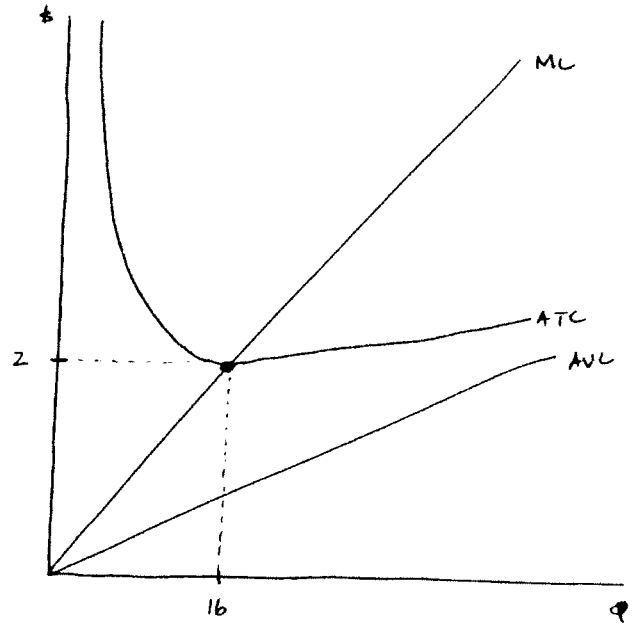
$$Q = K^{1/2} L^{1/2}$$

$$K^* = \omega^{1/2} r^{-1/2} Q \qquad L^* = \omega^{-1/2} r^{1/2} Q$$

$$C_{LR}(\omega, r, Q) = 2\,\omega^{1/2} r^{1/2} Q$$

$$C_{SR}(\omega, r, Q) = \omega \cdot \frac{Q^2}{\bar{K}} + r \cdot \bar{K}$$





$$C_{LR} = 2Q$$

$$C_{SR} = \frac{Q^2}{16} + 16$$

$$MC = \frac{Q}{8}$$

$$ATC = \frac{Q}{16} + \frac{16}{Q}$$

$$AVC = \frac{Q}{16}$$

i) CRS Technology → Inputs double, costs double; output doubles.

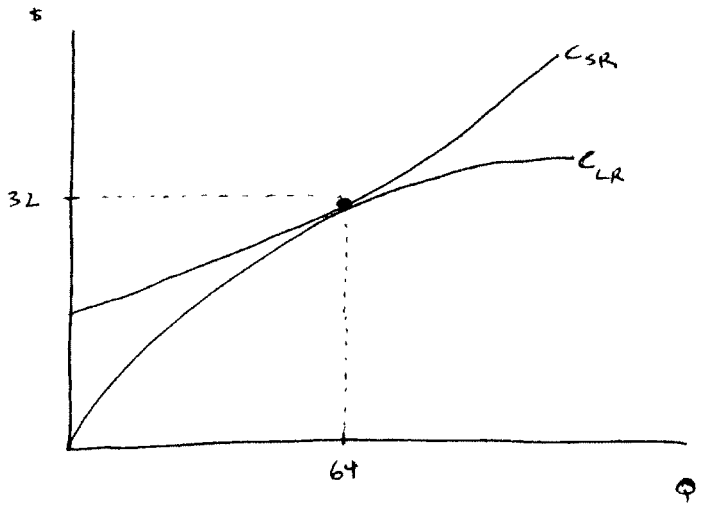
ii) After $Q = 16$ AVC dominates AFC.

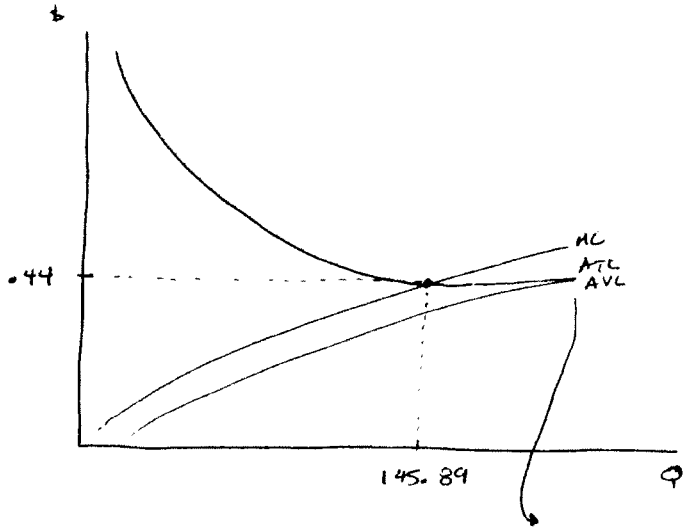
3, 8, 13, 18

$Q = K^{3/4} L^{3/4}$

$K^* = w^{1/2} r^{-1/2} Q^{2/3}$ $\qquad$ $L^* = w^{-1/2} r^{1/2} Q^{2/3}$

$C_{LR}(w, r, Q) = 2 w^{1/2} r^{1/2} Q^{2/3}$ $\qquad$ $C_{SR}(w, r, Q) = w \frac{Q^{4/3}}{\bar{K}} + r\bar{K}$





$C_{LR} = 2 \cdot Q^{2/3}$

$C_{SR} = \frac{Q^{4/3}}{16} + 16$

$MC = \frac{Q^{1/3}}{12}$

$AVC = \frac{Q^{1/3}}{16}$

$ATC = \frac{Q^{1/3}}{16} + \frac{16}{Q}$

ATC and AVC will never touch!

i) IRS – As inputs double, costs double; output more than doubles

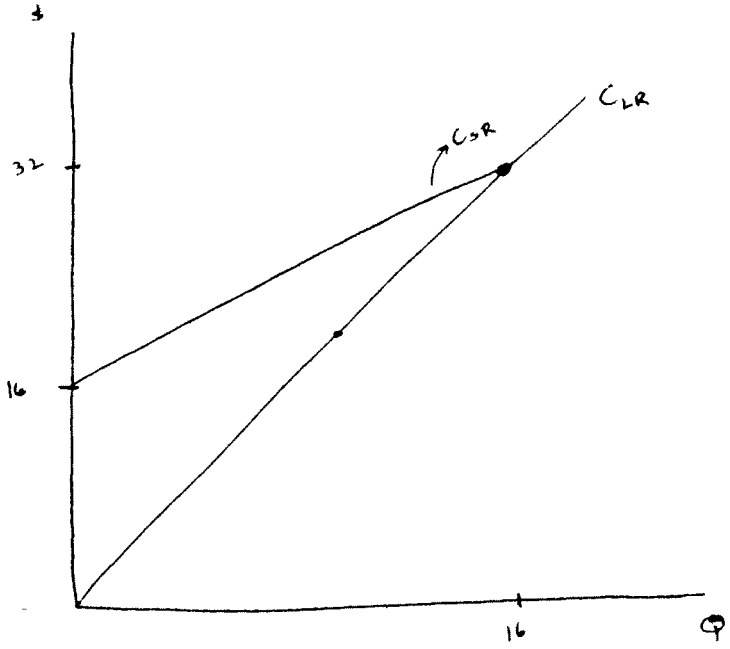
ii) Past $Q = 145.89$ AVC dominates AFC and ATC begins to rise.

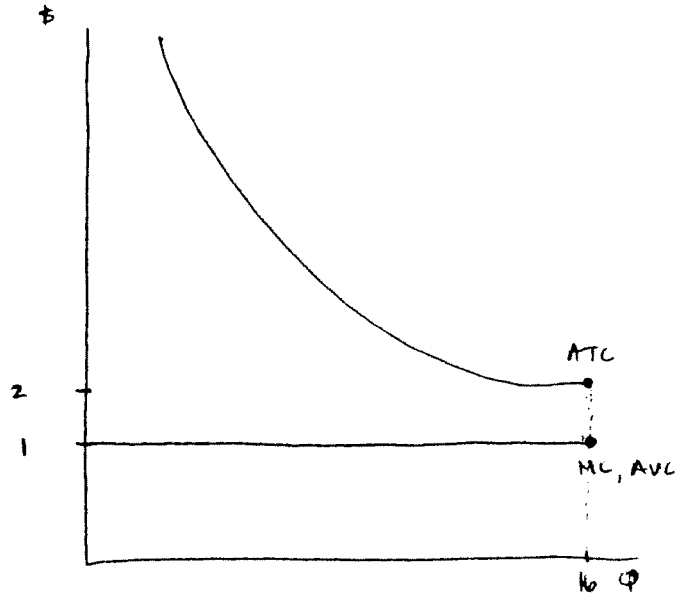
4, 9, 14, 19

$Q = \text{Min}\{K, L\}$ $\left.\begin{matrix} K = Q \\ L = Q \end{matrix}\right\}$ C.F. Ds

$C_{LR} = w \cdot Q + r \cdot Q$

$C_{SR} = w \cdot Q + r \cdot \bar{K}$ for $Q \leq \bar{K}$





$C_{LR} = 2Q$

$C_{SR} = Q + 16$

$MC = 1$

$AVC = 1$

$ATC = 1 + \frac{16}{Q}$

Note: In the SR, this firm is incapable of producing more than 16 units of output.

i) CRS Technology

ii) Dominated by AFC over entire domain $\Rightarrow 0 \leq Q \leq 16$

5, 10, 15, 20

$Q = 4 \cdot K + 2 \cdot L$

CEDs

$$L = \begin{cases} 0 & \text{if } \frac{w}{r} > \frac{1}{2} \\ [0, Q/2] & \text{if } \frac{w}{r} = \frac{1}{2} \\ Q/2 & \text{if } \frac{w}{r} < \frac{1}{2} \end{cases}$$

$$K = \begin{cases} 0 & \text{if } \frac{w}{r} < \frac{1}{2} \\ [0, Q/4] & \text{if } \frac{w}{r} = \frac{1}{2} \\ Q/4 & \text{if } \frac{w}{r} > \frac{1}{2} \end{cases}$$

Long run cost:

Since the production function exhibits CRS - find cheapest way of producing one unit, then multiply cost per unit by the total # of units produced.

If $Q = 1$

$K = 1/4$ $\quad r \cdot 1/4$

or Cost → So,

$L = 1/2$ $\quad w \cdot 1/2$

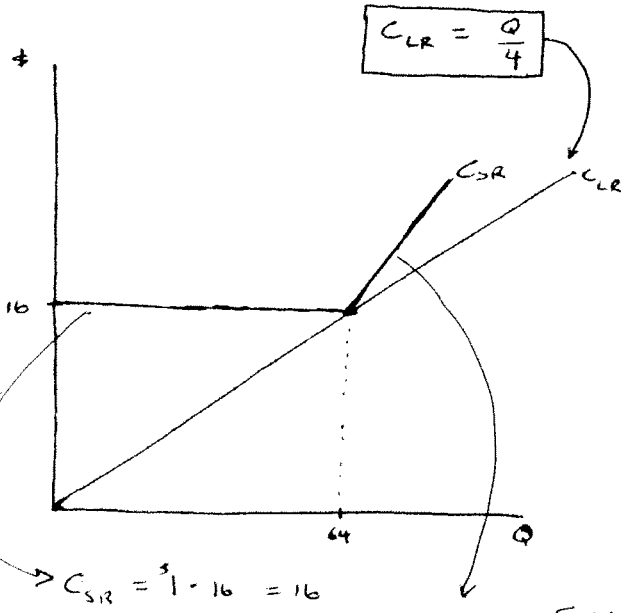
$$C_{LR} = \text{Min}\{r/4, w/2\} \cdot Q$$

$C_{SR} = r \cdot K$ if $Q < 4\bar{K}$

$C_{SR} = w \cdot \left[\frac{Q}{2} - 2\bar{K}\right] + r \cdot \bar{K}$ if $Q > 4\bar{K}$

Note: This firm can't/won't produce less than

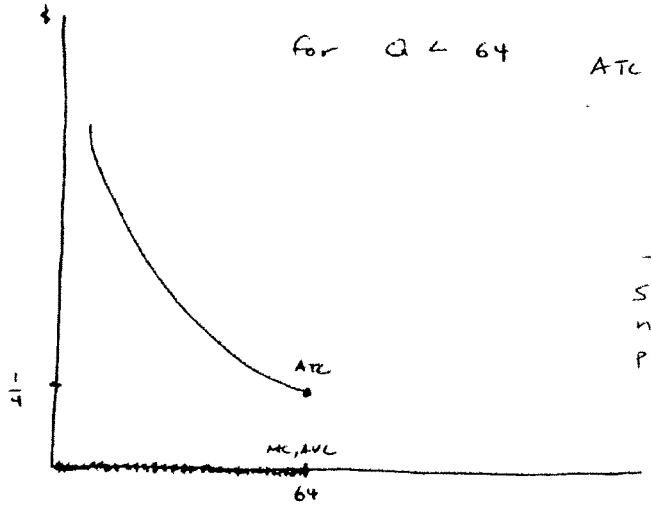
$Q = 64$ in the short run! $\Rightarrow Q = 4 \cdot 16 = 64$



$C_{SR} = 1\left[\frac{Q}{2} - 2(16)\right] + 16$

MC = 0 AVC = 0 ATC = AFC for $Q < 64$

$ATC = \frac{16}{Q}$



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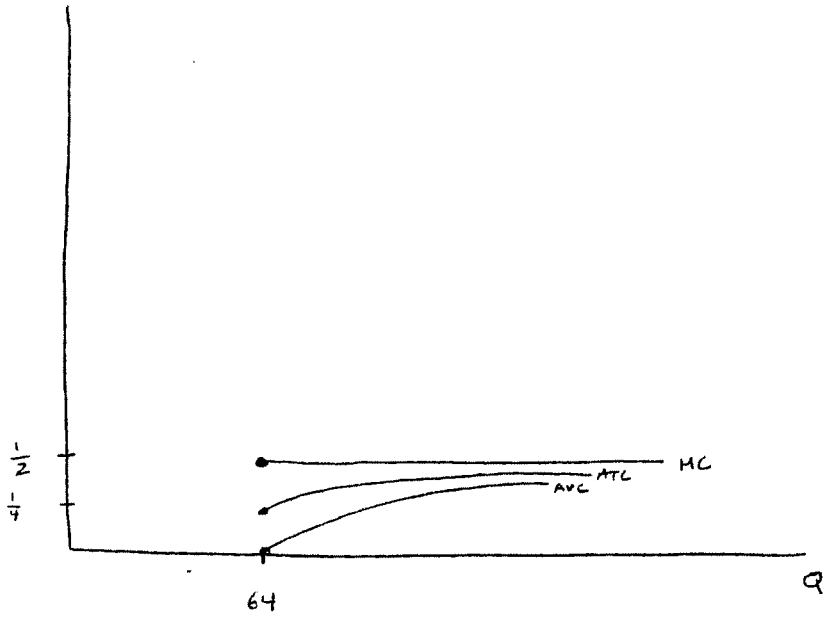

for $Q \geq 64$

$MC = \frac{1}{2}$

$AVC = \frac{\left[\frac{Q}{2} - 32\right]}{Q}$

$AVC = \frac{1}{2} - \frac{32}{Q}$

$ATC = \frac{1}{2} - \frac{16}{Q}$

i) CRS Technology.

ii) AFC dominates until $Q = 64$, AVC dominates after that

21

a) $TVC = 8Q^3 + 24Q$

$TFC = 48$

b) $ATC = 8Q^2 + 24 + \frac{48}{Q}$

$AVC = 8Q^2 + 24$

$AFC = \frac{48}{Q}$

$MC = 24Q^2 + 24$

c)