

Problem Set 8 - Answer Key

1) a)  $P = 100 - \frac{1}{10}Q$   
 $MR = 100 - \frac{2}{10}Q$

$MR = MC$  to maximize profit.

$$100 - \frac{2}{10}Q = 10$$

$$90 = \frac{2}{10}Q \quad Q = \frac{900}{2}$$

$Q = 450$   
 $P = 55$

Notice: This material may be protected by copyright law (Title 17 U.S. Code)

b)  $P = 100 - \frac{1}{10}Q$   
 $MR = 100 - \frac{2}{10}Q$   
 $MC = 4Q$

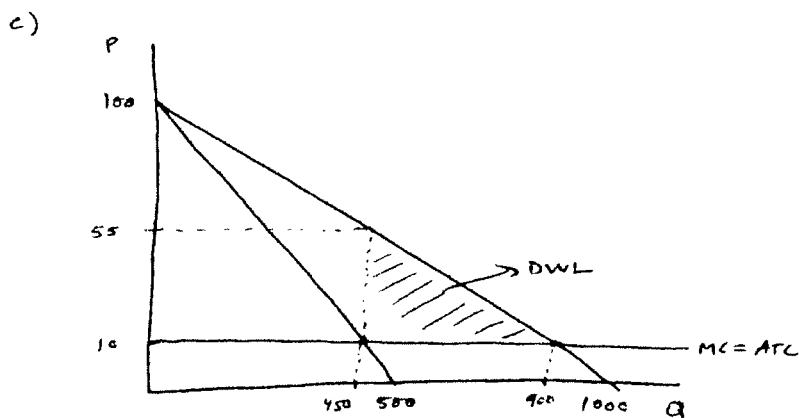
$MR = MC \rightarrow$  to maximize profit

$$100 - \frac{2}{10}Q = 4Q$$

$$1000 - 2Q = 40Q$$

$$42Q = 1000$$

$Q^* = 23.8$   
 $P^* = 97.62$



$$\pi = (P - ATC) \cdot Q^*$$

$$\pi = (55 - 10) \cdot 450$$

$$\pi = 20,250$$

$$DWL = 10,125$$

2

$$P = \frac{MC}{\left(1 - \frac{1}{|E_d|}\right)} \Rightarrow Z = \frac{MC}{1 - \frac{1}{5}} \quad Z = \frac{MC}{4/5}$$

$$Z \cdot 4/5 = MC \Rightarrow \boxed{MC = 5.60}$$

3

$$P = \frac{MC}{\left(1 - \frac{1}{|E_d|}\right)}$$

Let  $E_d = Z$  where  $0 < Z < 1$

Assume  $P \geq 0$

Assume  $MC \geq 0$

$$P = \frac{MC}{\frac{Z-1}{Z}} \Rightarrow \frac{MC}{P} = \frac{Z-1}{Z}$$

Since  $0 < Z < 1$ , then  $\frac{Z-1}{Z} < 0$ .

Therefore,

$$\frac{MC}{P} = \frac{Z-1}{Z} \quad \text{only if } P < 0 \text{ or}$$

$MC < 0$  (not both). This can't happen. (by assumption.)

Therefore  $E_d$  can't be less than 1.

4

$$\pi = \left[ 4 - \frac{1}{100}(q_1 + q_2) \right] (q_1 + q_2) - \left[ 10 + \frac{q_1^2}{10} \right] - \left[ 40 + \frac{q_2^2}{20} \right]$$

$$-\frac{\partial \pi}{\partial q_1} = 4 - \frac{2}{100}(q_1 + q_2) - \frac{q_1}{5} = 0 \Rightarrow 400 - 2(q_1 + q_2) - 20q_1 = 0$$

$$400 - 2q_2 - 22q_1 = 0$$

$$\boxed{q_2 = 200 - 11q_1}$$

$$\frac{\partial \pi}{\partial q_2} = 4 - \frac{2}{100}(q_1 + q_2) - \frac{q_2}{10} = 0 \Rightarrow 400 - 2(q_1 + q_2) - 10q_2 = 0$$

$$400 - 2q_1 - 12q_2 = 0$$

$$\boxed{q_1 = 200 - 6q_2}$$

$$q_2 = 200 - 11(200 - 6q_2) \rightarrow 200 - 2200 + 66q_2 = q_2$$

$$65q_2 = 2000$$

$$\boxed{q_2 = 30.8}$$

$$q_1 = 200 - 6(200 - 11q_1) \rightarrow 200 - 1200 + 66q_1 = q_1$$

$$65q_1 = 1000$$

$$\boxed{q_1 = 15.4}$$

$$P = 4 - \frac{1}{100}(30.8 + 15.4)$$

$$\boxed{P^* = 4 - .462 = \underline{3.54}}$$

$$\pi = (3.54)(30.8 + 15.4) - \left[ 10 + \frac{15.4^2}{10} \right] - \left[ 40 + \frac{30.8^2}{20} \right]$$

$$\pi = 163.55 - [33.72] - [87.43]$$

$$\boxed{\pi = 42.40}$$

7

c)  $P = 10 - \frac{Q}{50}$

$MR = 10 - \frac{2Q}{25}$

$MC = 2 \rightarrow 2 = 10 - \frac{2Q}{25}$

$Q = 200$   
 $P = 6$

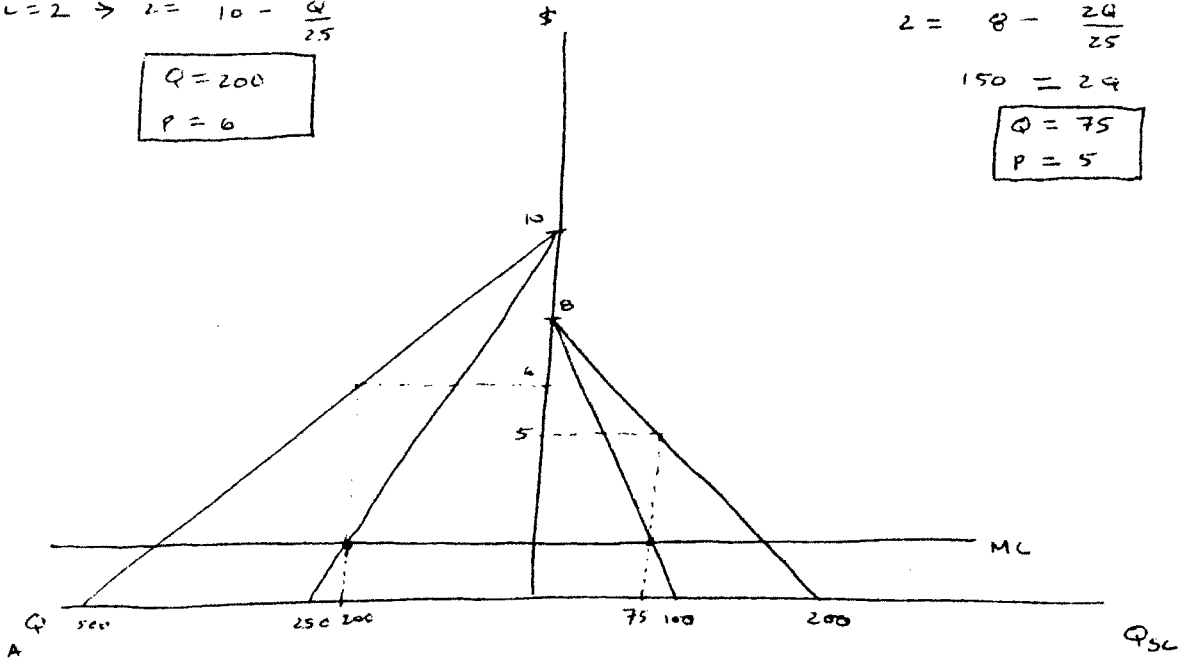
$P = 8 - \frac{4}{25}Q$

$MR = 8 - \frac{2Q}{25}$

$2 = 8 - \frac{2Q}{25}$

$150 = 2Q$

$Q = 75$   
 $P = 5$



b) Single Price Monopolist

$Q_A = 500 - 50P$

$Q_{Sc} = 200 - 25P$

$Q_M = 700 - 75P$

$P = 9.33 - \frac{1}{75}Q$

$MR = 9.33 - \frac{2}{75}Q$

$MC = 2$

$2 = 9.33 - \frac{2}{75}Q$

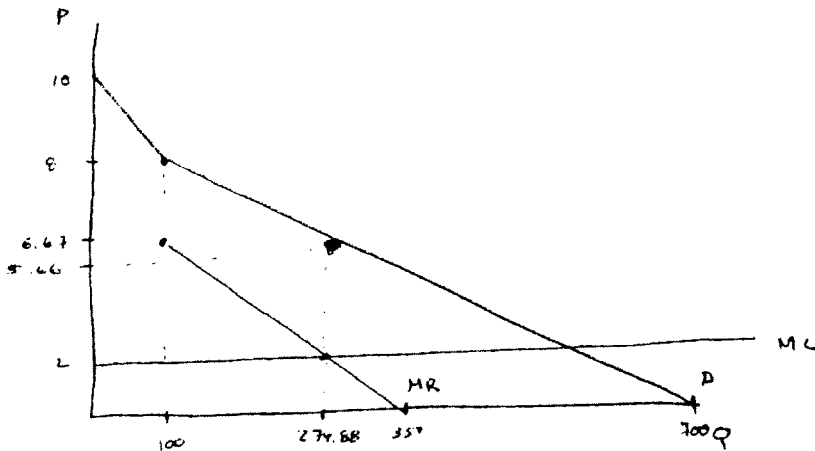
$7.33 = \frac{2}{75}Q$

$Q = 274.88$     $P = 5.66$

$\pi$  with single price

$\pi = (5.66)(274.88) - 100 + 2(274.88)$

$\pi = 906.06$



Continued

$\pi$  with 3<sup>rd</sup> Degree P.D

$$\pi = [200 \cdot 6] + [75 \cdot 5] - [100 + 2 \cdot 275]$$

$$\pi = 925$$

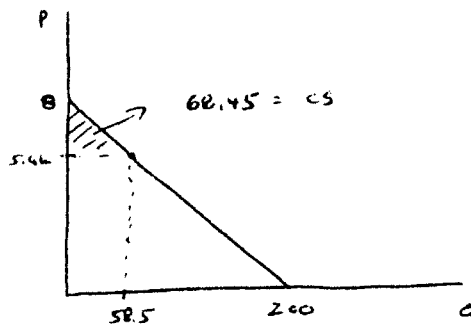
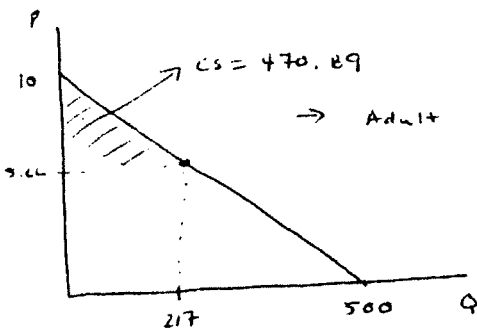
$$\rightarrow \text{Additional } \pi = 18.94$$

c) CS with P.D

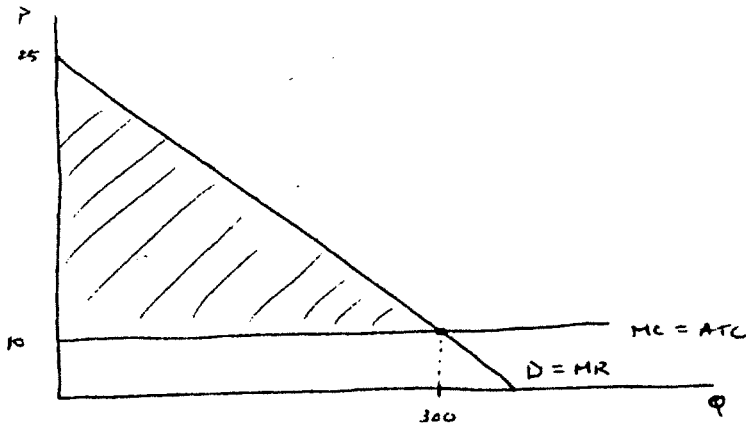
$$\text{Adults} \Rightarrow \frac{1}{2} \cdot 200 \cdot 4 = 400$$

$$\text{S.C.} \Rightarrow \frac{1}{2} \cdot 75 \cdot 3 = 112.50$$

CS without P.D



5



$$P = 25 - \frac{1}{20}Q$$

$$25 - \frac{1}{20}Q = 10$$

$$Q^* = 300$$

$$\pi = \frac{1}{2} \cdot 300 \cdot 15$$

$$\pi = 2250$$

6

1<sup>st</sup> Degree P.D does lead to an economically efficient outcome, a standard monopoly gen DWL.

8

$$\pi = [(1000 - 4Q_B)Q_B + (800 - 2Q_L)Q_L] - [10,000 + 5(Q_B + Q_L)^2]$$

$$\frac{d\pi}{dQ_B} = \underbrace{1000 - 8Q_B}_{MR} - \underbrace{10(Q_B + Q_L)}_{MC} = 0$$

$$\frac{d\pi}{dQ_L} = 800 - 4Q_L - 10(Q_B + Q_L) = 0$$

$$1000 - 8Q_B = 10Q_B + 10Q_L$$

$$1000 - 8Q_B = 800 - 4Q_L$$

$$200 + 4Q_L = 8Q_B$$

$$25 + \frac{1}{2}Q_L = Q_B$$

$$1000 - 10Q_L = 18\left(25 + \frac{1}{2}Q_L\right)$$

$$1000 - 10Q_L = 450 + 9Q_L$$

$$\begin{array}{l} Q_L = 29 \\ Q_B = 40 \end{array}$$

$$P_L = 742$$

$$P_B = 840$$

$$\pi = \$21,518 + \$33,600 - [10,000 + 5(69)^2]$$

$$\pi = \$55,118 - \$33,805 = \$21,313$$