Problem Set 9 – Answer Key

#1

$$\pi_A = \left[100 - \tfrac{1}{2}(Q_A + Q_Z)\right] Q_A - 10 Q_A$$

$$\frac{\partial \pi_A}{\partial Q_A} = 100 - Q_A - \tfrac{1}{2} Q_Z - 10 = 0 \Rightarrow \boxed{Q_A = 90 - \tfrac{1}{2} Q_Z}$$

$$\pi_Z = \left[100 - \tfrac{1}{2}(Q_A + Q_Z)\right] Q_Z - 10 Q_Z$$

$$\frac{\partial \pi_Z}{\partial Q_Z} = 100 - \tfrac{1}{2} Q_A - Q_Z - 10 = 0 \Rightarrow \boxed{Q_Z = 90 - \tfrac{1}{2} Q_A}$$

$$Q_A = 90 - \tfrac{1}{2}\left(90 - \tfrac{1}{2} Q_A\right) \qquad Q_A = 45 + \tfrac{1}{4} Q_A$$

$$4 Q_A = 180 + Q_A$$

$$3 Q_A = 180$$

$$\boxed{Q_A = 60 \\ Q_Z = 60} \qquad \boxed{P^* = 40}$$

Note:

#3

Take Zed's best response function from above.

$$Q_Z = 90 - \tfrac{1}{2} Q_A$$

Set up ACME's π function.

$$\pi = \left[100 - \tfrac{1}{2}\left((90 - \tfrac{1}{2} Q_A) + Q_A\right)\right] Q_A - 10 Q_A$$

$$\frac{\partial \pi}{\partial Q_A} = 100 - 45 + \tfrac{1}{2} Q_A - Q_A - 10 = 0 \Rightarrow \boxed{Q_A = 90 \\ Q_Z = 45} \qquad \boxed{P^* = 32.50}$$

#2

$$\pi = \left[100 - \tfrac{1}{2}(Q_A + Q_2)\right] Q_A + Q_2 - 10Q_A - 10Q_2$$

$$\frac{\partial \pi}{\partial Q_A} = 100 - (Q_A + Q_2) - 10 = 0$$

$$Q_A = 90 - Q_2$$

$$\frac{\partial \pi}{\partial Q_2} = 100 - (Q_A + Q_2) - 10 = 0$$

$$Q_2 = 90 - Q_A$$

$$Q_A = 90 - (90 - Q_A) \quad \Rightarrow \quad Q_A = Q_A$$

Not a unique solution!

Since $MC_A = MC_2 = 10$ use either "plant".

$MR = 100 - Q \qquad MC = 10$

$Q^* = 90 \Rightarrow$ Use either plant but produce 90 units.

Assume an even split.

$$Q_A = 45 \qquad Q_2 = 45 \qquad P^* = 55$$

#4

Model	Q_A	Q_Z	P	π_A	π_Z
Cournot	60	60	40	1800	1800
Collusion	45	45	55	2025	2025
Stackleberg	90	45	32.50	2025	1012.50

#5

Zed's π Function

$$\pi = P \cdot Q_Z - \frac{Q_Z^2}{2}$$

$$\frac{d\pi}{dQ_Z} = P - Q_Z = 0 \qquad Q_Z = P$$

Residual Demand for ACME ⇒

$$R(P) = Q_M(P) - Q_S \quad \rightarrow \text{Zed's supply function.}$$

$$R(P) = 200 - 2P - P$$

$$Q = 200 - 3P \qquad P = 66.7 - \frac{1}{3}Q$$

$$MR = 66.7 - \frac{2}{3}Q$$

ACME ⇒ π Max.

$$66.7 - \frac{2}{3}Q = 2 \qquad 200 - 2Q = 6$$

$$194 = 2Q$$

$Q_Z = 34.4$

$Q_A = 97 \rightarrow P = 34.4$

#6

$\pi_A = \$3142.80$ $\pi_Z = \$591.68$