

Quiz 4

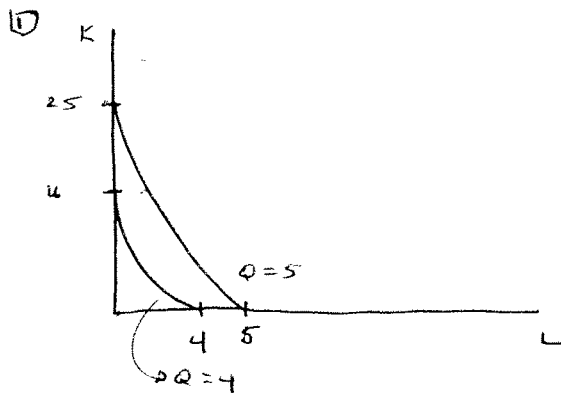
Instructions:

- You may not consult any outside materials. You may not consult with your classmates.
- You **may** use a calculator.
- You have twenty-five minutes to complete the quiz.
- If you use additional sheets of paper, then please staple the sheets to the quiz. Also, please be sure to cut any whiskers from the additional sheets you use.

ACME produces widgets using the following production function:

$$Q = f(l, k) = K^{1/2} + L$$

1. (6 points) Graph at least two isoquants for this production function. Be sure to label the output levels associated with each isoquant. Be sure to place labor on the horizontal axis.
2. (5 points) Does this production function exhibit increasing, constant, or decreasing returns to scale? How do you know this? Show your work.
3. (5 points) Does this production function exhibit diminishing marginal productivity to capital? How do you know this? Show your work.
4. (4 points) Calculate the elasticity of substitution for this production function. Use the isoquant for four units of output, and use the input bundle with one unit of labor and nine units of capital as your initial (old) bundle and the input bundle with four units of capital and two units of labor as your final (new) bundle.



2

K	L	Q
2	2	$\sqrt{2} + 2 = 3.42$
4	4	6

DRS !

3

$$\frac{\partial f}{\partial K} = \frac{1}{2} K^{-1/2}$$

$$\frac{\partial^2 f}{\partial K^2} = -\frac{1}{4} K^{-3/2} < 0 \Rightarrow MP_K \text{ is diminishing.}$$

4

$$\sigma = \frac{\% \Delta \frac{K}{L}}{\% \Delta \text{MRTS}} = \frac{\frac{\frac{K_2}{L_2} - \frac{K_1}{L_1}}{\frac{K_1}{L_1}}}{\frac{2K_2^{1/2} - 2K_1^{1/2}}{2K_1^{1/2}}}$$

$$\text{MRTS} = \frac{MP_L}{MP_K} = \frac{1}{\frac{1}{2} \cdot K^{-1/2}}$$

$$\sigma = \frac{\frac{\frac{4}{2} - \frac{9}{1}}{\frac{9}{1}}}{\frac{2(4)^{1/2} - 2(9)^{1/2}}{2(9)^{1/2}}} = \frac{-\frac{7}{9}}{-\frac{2}{6}}$$

$$\sigma = \frac{42}{18} = \frac{14}{6} = 2\frac{1}{3}$$