Economics 202
Dr. Smith

Name: Key

Pledge: _____

Quiz 9

Instructions:

- You may not consult any outside materials. You may not consult with your classmates.
- You **may** use a calculator.
- You have twenty-five minutes to complete the quiz.

Assume that Specialized and Bianchi are the only two firms in the bicycle industry. Furthermore, assume that the firms have the following cost functions:

Specialized: $C(Q) = \frac{1}{2}Q_S^2$ (Industry Leader, if applicable)
Bianchi: $C(Q) = Q_B^2$

The market demand for bikes is given by the following equation:

$$P = 1000 - Q_T \qquad \text{where} \qquad Q_T = Q_S + Q_B$$

Based on the information above, fill in the following table. You must show your work in order to receive full credit.

Duopoly Model	Specialized's Quantity Produced	Specialized's Profit 112,500	Bianchi's Quantity Produced	Bianchi's Profit 61,250	Market Price
Stackelberg	300	[redacted]	175	[redacted]	[redacted] → 525
Price Leadership	286	95,238	238	56,644	476

Stackelberg: Follower: $\pi = (1000 - (Q_S + Q_B))Q_B - Q_B^2$

$\pi = 525 \cdot 175 - 175^2$

$\pi = 70{,}000$

$\frac{\partial \pi}{\partial Q_B} = 1000 - Q_S - 2Q_B - 2Q_B = 0$

$Q_B = 250 - \frac{1}{4}Q_S$

Leader: $\pi = \left(1000 - \left(Q_S + 250 - \frac{1}{4}Q_S\right)\right)Q_S - \frac{1}{2}Q_S^2$

$\pi = 525 \cdot 300 - \frac{1}{2}(300)^2$

$\pi = 127{,}500$

$\frac{\partial \pi}{\partial Q_S} = 1000 - 2Q_S - 250 + \frac{1}{2}Q_S - Q_S = 0$

$750 = 2.5Q_S$

$Q_S = 300 \qquad Q_B = 175$

$P = 525$

Price Leadership:

Follower $\Rightarrow$ $\pi = P \cdot Q_B - Q_B^2 = 0$

$$\frac{\partial \pi}{\partial Q_B} = P - 2Q_B = 0$$

$$\boxed{Q_B = \frac{P}{2}} \Rightarrow \boxed{P = 2Q_B}$$

follower's supply

Residual Demand $\Rightarrow$ $R(P) = [1000 - P] - \left[\frac{P}{2}\right]$

$$R(P) = 1000 - \frac{3}{2}P$$

$$P = \frac{2000}{3} - \frac{2}{3}Q$$

Leader:

$$MR = MC$$

$$\frac{2000}{3} - \frac{4}{3}Q = Q$$

$$2000 - 4Q = 3Q$$

$$2000 = 7Q$$

$$\boxed{\begin{aligned} Q_S &= 285.7 = 286 \\ P^* &= 476 \\ Q_B &= 238 \end{aligned}}$$

$\pi_B = 238 * 476 - 238^2 = 56,644$

$\pi_S = 286 * 476 - \frac{1}{2} 286^2 = 95,238$