# Compensated vs. Uncompensated Demand

If Sam has the utility function $U = X_1 X_2$, then we can derive her compensated and uncompensated demand functions.

Uncompensated $\Rightarrow$ Max $U = x_1 x_2$ s.t. $P_1 X_1 + P_2 X_2 = I$

Compensated $\Rightarrow$ Min $P_1 X_1 + P_2 X_2$ s.t. $X_1 \cdot X_2 = \bar{U}$

In class we found these demand functions.

<u>Uncompensated</u>

$$X_1 = \frac{1}{2} \cdot \frac{I}{P_1}$$

$$X_2 = \frac{1}{2} \cdot \frac{I}{P_2}$$

<u>Compensated</u>

$$X_1^{comp} = \sqrt{\frac{P_2}{P_1} \bar{U}}$$

$$X_2^{comp} = \sqrt{\frac{P_1}{P_2} \bar{U}}$$

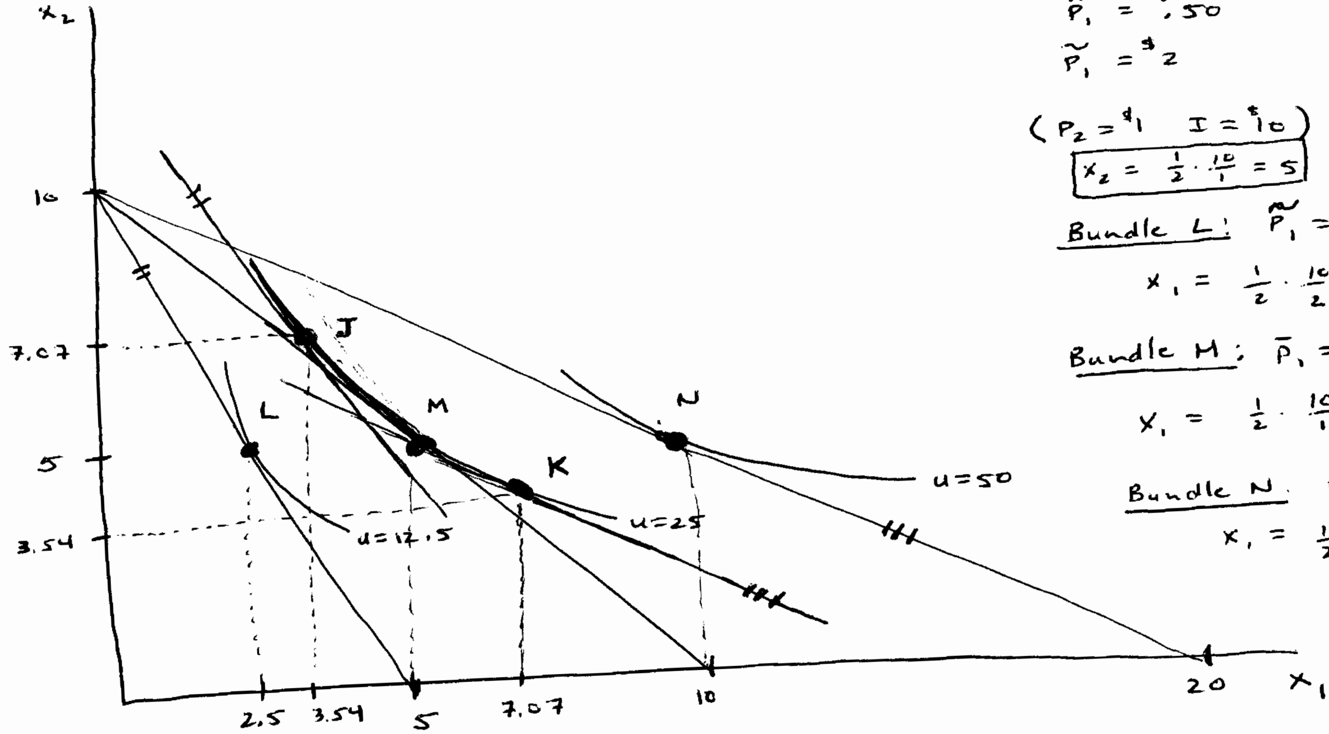
So, what's the difference between uncompensated and compensated demand?

To understand the answer to this question let's look at a pair of graphs.... (see page 2)

Let's let $I = \$10$ $P_2 = \$1$. If $P_1$ varies, then we derive (graphically) the uncompensated demand for good 1. (Note: We don't <u>need</u> to do this, for we know the uncompensated demand function would be

$$X_1 = \frac{1}{2} \cdot \frac{\$10}{P_1} \Rightarrow X_1 = \frac{5}{P_1}$$ )

Figure One




## Uncompensated Demand

Let $\bar{P}_1 = \$1$

$\hat{P}_1 = \$.50$

$\tilde{P}_1 = \$2$

$(P_2 = \$1 \quad I = \$10)$

$\boxed{x_2 = \frac{1}{2} \cdot \frac{10}{1} = 5}$

Bundle L: $\tilde{P}_1 = \$2$

$x_1 = \frac{1}{2} \cdot \frac{10}{2} = 2.5$

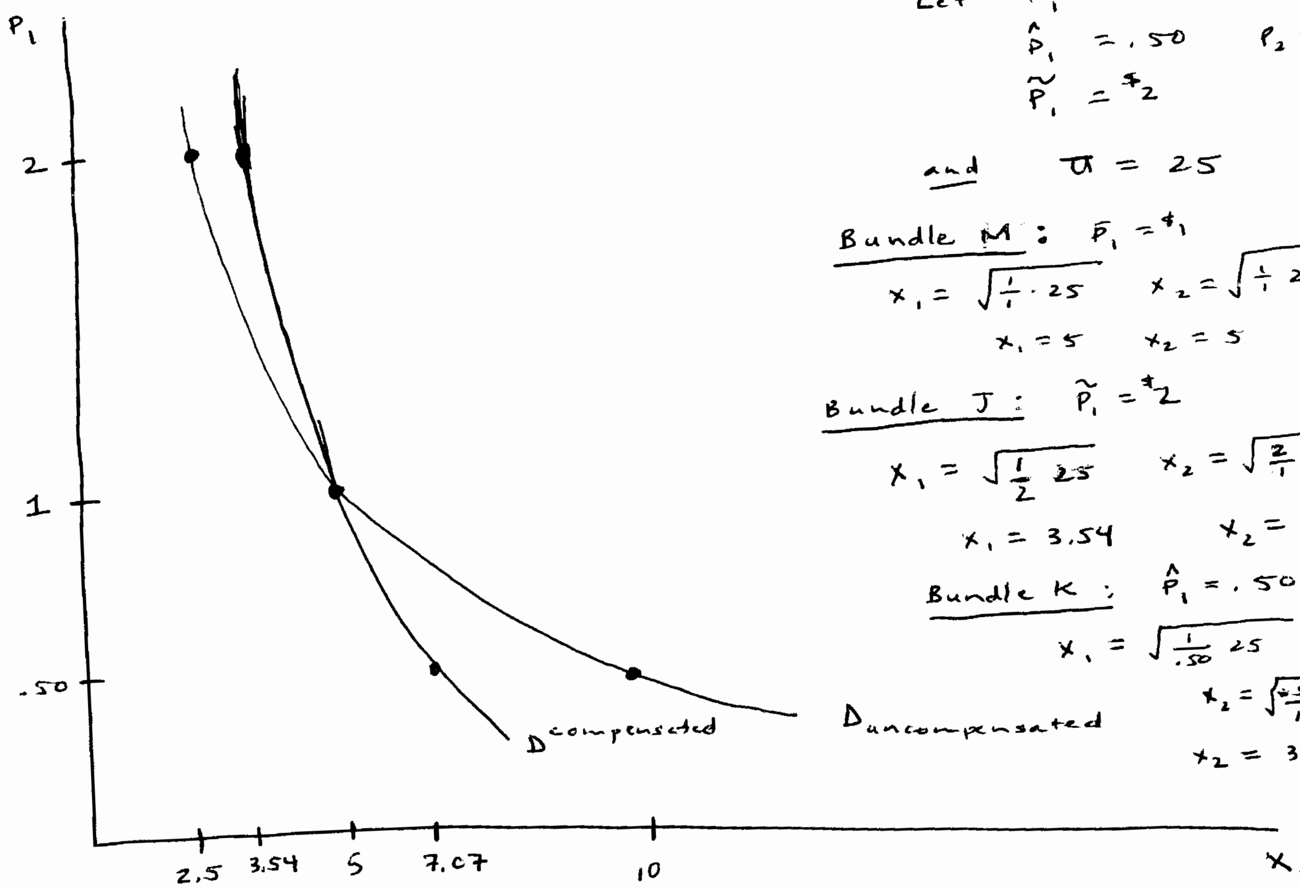
Bundle M; $\bar{P}_1 = \$1$

$x_1 = \frac{1}{2} \cdot \frac{10}{1} = 5$

Bundle N. $\hat{P}_1 = .50$

$x_1 = \frac{1}{2} \cdot \frac{10}{.50} = 10$

Figure Two




## Compensated Demand

Let $\bar{P}_1 = \$1$

$\hat{P}_1 = .50 \qquad P_2 = \$1$

$\tilde{P}_1 = \$2$

and $\bar{U} = 25$

Bundle M: $\bar{P}_1 = \$1$

$x_1 = \sqrt{\frac{1}{1} \cdot 25} \qquad x_2 = \sqrt{\frac{1}{1} \, 25}$

$x_1 = 5 \qquad x_2 = 5$

Bundle J: $\tilde{P}_1 = \$2$

$x_1 = \sqrt{\frac{1}{2} \, 25} \qquad x_2 = \sqrt{\frac{2}{1} \, 25}$

$x_1 = 3.54 \qquad x_2 = 7.07$

Bundle K: $\hat{P}_1 = .50$

$x_1 = \sqrt{\frac{1}{.50} \, 25} = 7.07$

$x_2 = \sqrt{\frac{.50}{1} \cdot 25}$

$x_2 = 3.54$

If we find three optimal bundles (L, M, N) we may graph those bundles on a budget line/indifference curve graph. Then, we may translate that information to a second set of axes to create our uncompensated demand curve. (Again, this is the same curve we would have graphed if we had plotted points for the equation $x_1 = \frac{5}{P_1}$.)

Figure One, page 2

Now, suppose we want to compare this uncompensated demand curve to <u>a</u> compensated demand. We must pick <u>a</u> utility level. Let's choose $\bar{U} = 25$. This is the indifference curve that bundle M sits on. To obtain additional points on the compensated demand we keep $P_2$ fixed (at \$1) and $\bar{U}$ fixed at 25 and we let $P_1$ vary. If we choose the same values for $\hat{P}_1$ and $\tilde{P}_1$, then we obtain the results seen in figures 1 and 2.

Q/ Why is the compensated demand steeper than the uncompensated demand?

A/ $X_1$ is a normal good. (Make sure you can show this by graphically deriving the substitution and income effects.) Since $X_1$ is normal, when Sam's income rises her consumption of $X_1$ increases.

When the price of good 1 falls, Sam feels two effects – the change in relative prices (the substitution effect), and a change in real income (the income effect). The uncompensated demand captures <u>both</u> effects. However, the compensated demand forces Sam <u>to ignore the income effect.</u> Sam's utility (her real "income") is held constant!

Therefore, when $P_1$ falls from \$1 to \$.50 Sam's uncompensated demand at $P = .50$ is greater than her compensated demand at $P = .50$ → the uncompensated demand includes the income effect!

→ Make sure you can explain why Sam's uncompensated demand is less than her compensated demand (at $P = \$2$) when the price increases from \$1 to \$2!

→ What if $X_1$ were <u>an</u> <u>inferior</u> good??