Lecture

- Risk and Uncertainty
- Expected Utility
- Insurance

So far, our analysis has focused on choices between certain outcomes.

Ex.

2 pizza slices, 2 cokes

1 pizza slice, 3 cokes

Some "choices" involve uncertainty.

Ex. Should I buy insurance?

- Hurricane Insurance
- Life Insurance
- Auto Insurance

Answer to this question involves an individual's preferences.

To understand this more fully let's consider an example...

There are two states of nature – "good" and "bad".

In our example, we'll focus on driving.

π_1 = probability of "bad" state

π_2 = probability of "good" state

$\pi_1 + \pi_2 = 1$

Asset = Volvo Station Wagon = 35,000

"good" state – no wreck. "Bad" State – wreck. \$10,000 of damage

The consumer "chooses" between probability distributions.

<u>Choice 1</u> : No insurance

State 1 : \$25,000 State 2: \$35,000

<u>Choice 2</u> : \$5,000 insurance policy that costs \$50

State 1: \$35,000 – \$50 – \$10,000 + \$5,000 = \$29,950

State 2: \$35,000 – \$50 = \$34,950

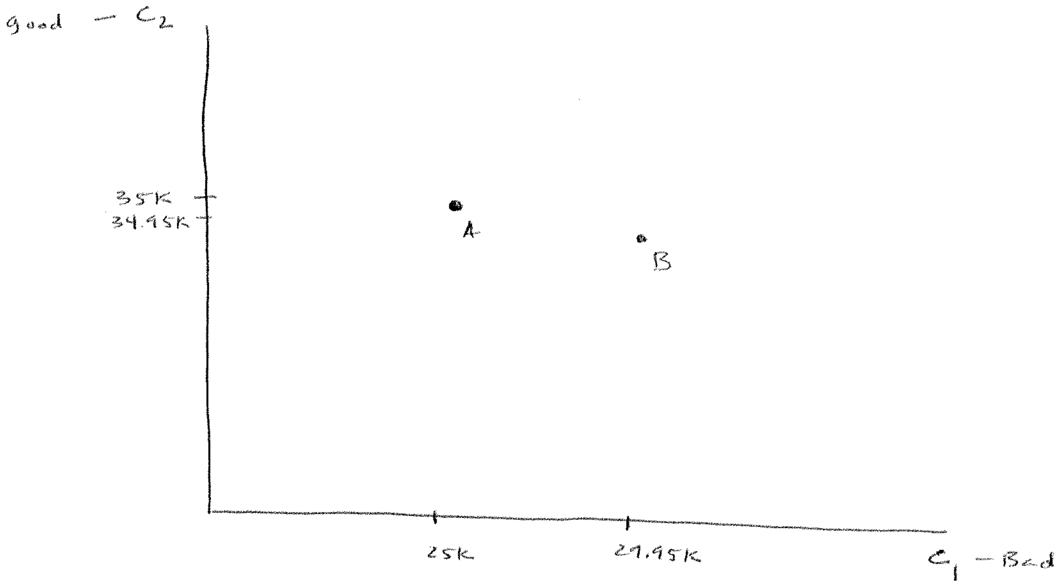
More generally :

State 1 : give up: γK get: K

State 2: give up: γK get: ϕ

K = dollars of insurance

γ = Premium for Insurance



Which option will the consumer choose?

Depends on preferences!

What type of utility functions will we use?

<u>Expected Utility</u>

The general form we'll use:

$$u(\pi_1, \pi_2, c_1, c_2) = \pi_1 V(c_1) + \pi_2 V(c_2)$$

$V(c_i)$ = a function that turns consumption into utility.

Ex.

$$V(c_1) = c_1$$

$$V(c_i) \Rightarrow \quad \ln(c_1)$$

$$\sqrt{c_1}$$

$$c_1^2$$

$\longrightarrow$ Making use of these

Using this information we get some expected utility functions

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1 + \pi_2 c_2$$

↳ "Expected Value"

Ex.

$\pi_1 = .01$ $\quad\quad \pi_2 = .99$

$c_1 = 25{,}000$ $\quad\quad c_2 = 35{,}000$

$$u(25{,}000, 35{,}000, .01, .99) = (.01)\,25{,}000 + (.99)\,35{,}000$$

$$= 250 + 34{,}650$$

$$EV = 34{,}900$$

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1^{1/2} + \pi_2 c_2^{1/2}$$
$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 \ln c_1 + \pi_2 \ln c_2$$

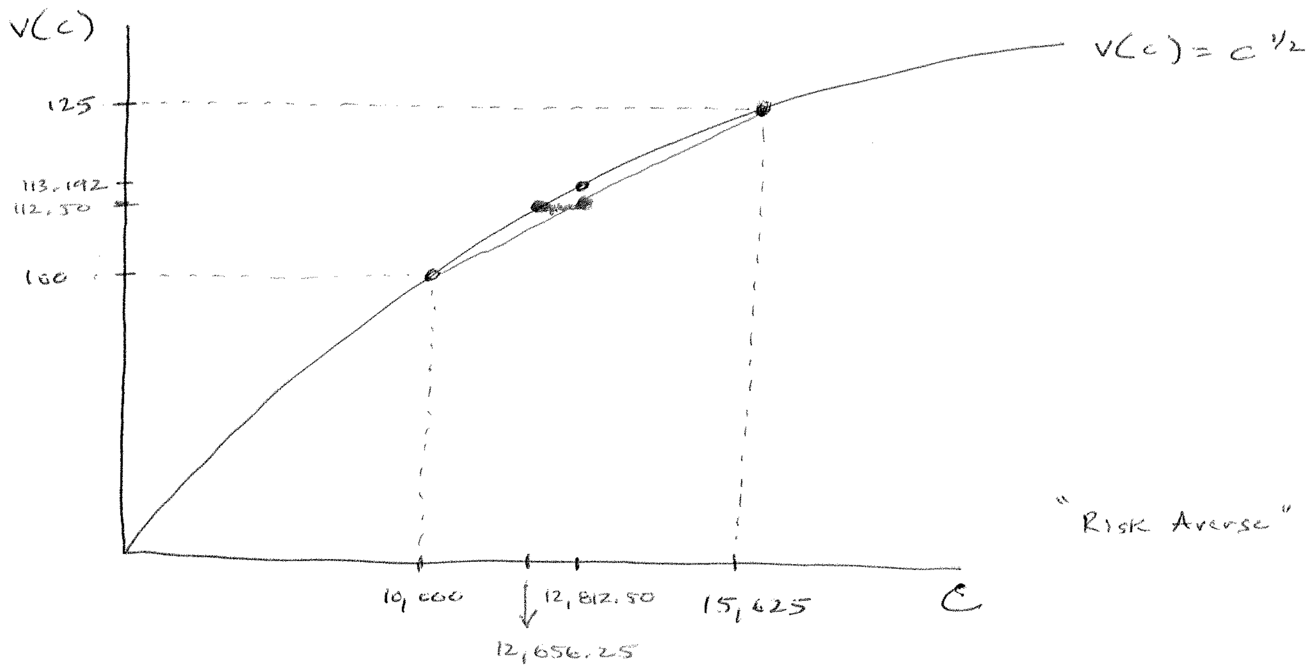
$$u = \pi_1 c_1^2 + \pi_2 c_2^2$$

The shape of $v(c_i)$ determines the shape of the consumer's indifference curves!

First, though, let's consider what the shape of the v functions look like — and what the shapes mean.

Ex.

$V(c) = c^{1/2}$



"good" State 2 : $\pi_1 = .50$ 15,625

"bad" State 1 . $\pi_2 = .50$ 10,000

12,812.50 − 12,626.25 = \$156.25 → Would give this up to not pay the lottery.

If forced to play the lottery —

Average Utility ⇒ $\pi_1 \cdot V(c_1) + \pi_2 V(c_2)$

If allowed to take the sure thing —

Utility of Expected Value ⇒ $V(\pi_1 c_1 + \pi_2 c_2)$

In this case, U of EV > Average Utility

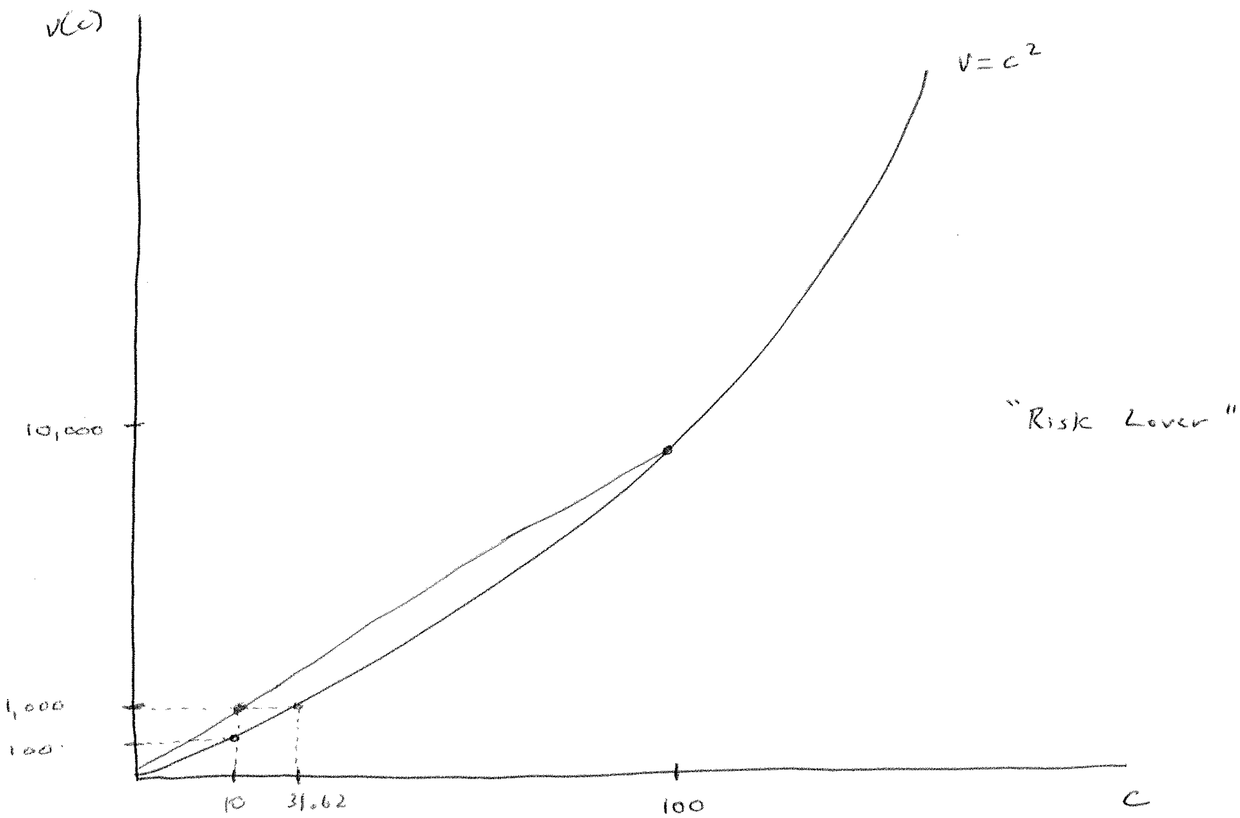
Risk Averse — Prefers a sure thing!

Ex. 2

$V(c) = c^2$

"good" State 2 $\rightarrow$ $\pi_1 = .10$ \$100

"Bad" State 1 $\rightarrow$ $\pi_2 = .90$ \$0



Average Utility $\Rightarrow$ $\pi_1 \cdot V(c_1) + \pi_2 \cdot V(c_2)$

$= 0 + 1{,}000 = 1{,}000$

Utility of Average $\Rightarrow$ $V(\pi_1 c_1 + \pi_2 c_2)$

$= V(0 + .10(100))$

$= V(10) = 100$

Needs 21.62 to agree not to pay the lottery.

Who will buy insurance ?

	State 1	State 2
No insurance	25,000	35,000
Insurance ↳ \$5000 (\$50 premium)	29,950	34,950

Risk Averse $\quad v(c) = \sqrt{c}$

None: $\quad u = .01\sqrt{25K} + .99\sqrt{35K} = 186.79$

With: $\quad u = .01\sqrt{29{,}950} + .99\sqrt{34{,}950} = \boxed{186.81}$

Risk Neutral $\quad v(c) = c$

None : $u = .01(25{,}000) + .99(35{,}000) = \boxed{34{,}900}$

with : $u = .01(29{,}950) + .99(34{,}950) = \boxed{34{,}900}$

Risk Lover $\quad v(c) = c^2$

None : $\quad u = .01(25K)^2 + .99(35K)^2 = \boxed{1219 \times 10^6}$

with : $\quad u = .01(29{,}950)^2 + .99(34{,}950)^2 = 1218 \times 10^6$

How much insurance will a consumer buy?

$\rightarrow \; \gamma = \pi_1$

→ If offered "actuarily fair" insurance, then Risk Averse consumers will always fully insure!

→ In our example the R. Averse consumer would like to buy 10,000 of insurance.

→ Risk Neutral → indifferent to insurance.

→ Risk Lover → won't insure!

Applications

— Diversifying your portfolio

— Keep expected payout the same, but produce the result with certainty.

— Insurance

— Diversify the insurance pool.

Ex. Health Insurance

Hurricane Insurance