Homework #4
Chain Rule for Partial Derivatives

1. Use the chain rule for partial derivatives to compute $\frac{dw}{dt}$ for each of the following sets of functions:

   (a) $w = e^{xy}$, $\quad x = 3t - 1$, and $y = t^2 - 3t + 2$.

   (b) $w = \ln(x^2 + y^2)$, $\; x = e^{-t}$ and $y = \sin(t)$.

2. Suppose that the pressure of a gas as a function of time is modeled by $P = 2 - e^{-0.5t}$ and the temperature is modeled by $T = 300 - 20e^{-0.1t}$ Use the chain rule for partial derivatives to find the rate of change in the volume of the gas with respect to time, assuming the ideal gas law applies.

3. *Bacillus* bacteria are shaped like a cylinder with a hemispherical cap on each end.

   (a) Look up the formulas for surface area and volume for the three parts of the bacteria to create a mathematical model for the bacteria shape.

   (b) Find the rate of change in the surface area and volume of the *bacillus* cell with respect to the length of the cylindrical portion, and with respect to the radius of the cylindrical portion.

   (c) Suppose that as the *bacillus* cell grows, the radius of the cylinder is given as a function of time as $r = 0.1 - 0.09e^{-t}$, and the length of the cylinder is given as a function of time as $L = 0.8 - 0.72e^{-2t}$. Use the chain rule for partial derivatives to find the rate of change in surface area and volume with respect to time.