Assign #12 §10.5 # 2, 10, 16

2) $A=\begin{bmatrix} 7 & 4 \\ -3 & -1 \end{bmatrix}$ $\det(A-\lambda I)=\det\begin{bmatrix} 7-\lambda & 4 \\ -3 & -1-\lambda \end{bmatrix}=(7-\lambda)(-1-\lambda)-(-12)=-7-6\lambda+\lambda^2+12$

$=\lambda^2-6\lambda+5=(\lambda-5)(\lambda-1)$

eigenvalues: $\det(A-\lambda I)=0 \Rightarrow \lambda=5$ and $\lambda=1$

$\lambda=5$: $(A-\lambda I)\vec{x}=\vec{0} \Rightarrow \left[\begin{array}{cc|c} 2 & 4 & 0 \\ -3 & -6 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$ x_2 free, $x_1=-2x_2$ e.g. $\vec{x}=\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$\lambda=1$: $\left[\begin{array}{cc|c} 6 & 4 & 0 \\ -3 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2/3 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$ x_2 free, $x_1=-\frac{2}{3}x_1$ e.g. $\vec{x}=\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

10) $A=\begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix}$ $\det(A-\lambda I)=\det\begin{bmatrix} 7-\lambda & -2 \\ 2 & 3-\lambda \end{bmatrix}=(7-\lambda)(3-\lambda)-(-4)=21-10\lambda+\lambda^2+4$

$=\lambda^2-10\lambda+25=(\lambda-5)^2$

eigenvalue: $\det(A-\lambda I)=0 \Rightarrow \lambda=5$

$(A-5I)\vec{x}=\vec{0} \Rightarrow \left[\begin{array}{cc|c} 2 & -2 & 0 \\ 2 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$ x_2 free, $x_1=x_2$ e.g. $\vec{x}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

16) $A=\begin{bmatrix} .95 & .03 \\ .05 & .97 \end{bmatrix}$ a) Find "proportional" equilibrium of size 10,000 $\Rightarrow$ Find e-vector

b) Find factor by which grows or declines $\Rightarrow$ Find e-value

$\det(A-\lambda I)=(.95-\lambda)(.97-\lambda)-.0015$ Set $\det(A-\lambda I)=0 \rightarrow \lambda=.92$ or $\lambda=1$

$\lambda=.92$ $\left[\begin{array}{cc|c} .03 & .03 & 0 \\ .05 & .05 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$ x_2 free, $x_1=-x_2$ e.g. $\vec{x}=\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ Can't be population b/c of negative

$\lambda=1$ $\left[\begin{array}{cc|c} -.05 & .03 & 0 \\ .05 & -.03 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3/5 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$ x_2 free, $x_1=\frac{3}{5}x_2$ e.g. $\vec{x}=\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ Want to add to 10,000 so $\vec{x}=\begin{bmatrix} 3750 \\ 6250 \end{bmatrix}$

b) "Growth" factor is 1 $\Rightarrow$ stable population