Assign #9 AR 2.1.7, 2.2.5 GRL §10.3 #6, 31, 32, 33, 36, 48

2.1.7 See Excel

2.2.5 $P = \begin{pmatrix} .2 & 3 \\ .3 & .5 \end{pmatrix} \Rightarrow \begin{aligned} I_{t+1} &= .2 I_t + 3 M_t \\ M_{t+1} &= .3 I_t + .5 M_t \end{aligned}$

a) 3 births per adult per time step

b) $.5 M_t$ in eqn for M_{t+1} indicates 50% survival, so 50% die

c) $.2 I_t$ in eqn for I_{t+1} is % of immature at one timestep that are still immature at next timestep.

d) 30% of immature individuals at one time step are mature at next timestep.

6) $3A - 10B = \begin{bmatrix} -6 & 12 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} -60 & 20 \\ 40 & 0 \end{bmatrix} = \begin{bmatrix} 54 & -8 \\ -40 & 9 \end{bmatrix}$

31) a) $AB = \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 16 & 22 \\ 7 & 19 \end{bmatrix}$ b) $BA = \begin{bmatrix} -2 & 1 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 0 & 30 \end{bmatrix}$

c) No! d) $AB \neq BA$, in general

32) $(PX)T = \left(\begin{bmatrix} m & n \\ p & q \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right)\begin{bmatrix} r & s \\ t & u \end{bmatrix} = \begin{bmatrix} mx+nz & my+nw \\ px+qz & py+qw \end{bmatrix}\begin{bmatrix} r & s \\ t & u \end{bmatrix}$

$= \begin{bmatrix} mxr + nzr + myt + nwt & mxs + nzs + myu + nwu \\ pxr + qzr + pyt + qwt & pxs + qzs + pyu + qwu \end{bmatrix}$

$P(XT) = \begin{bmatrix} m & n \\ p & q \end{bmatrix}\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} r & s \\ t & u \end{bmatrix}\right) = \begin{bmatrix} m & n \\ p & q \end{bmatrix}\begin{bmatrix} xr + yt & xs + yu \\ zr + wt & zs + wu \end{bmatrix}$

$= \begin{bmatrix} mxr + myt + nzr + nwt & mxs + myu + nzs + nwu \\ pxr + pyt + qzr + qwt & pxs + pyu + qzs + qwu \end{bmatrix}$

$(PX)T = P(XT)$

33) $P(X+T) \stackrel{?}{=} PX + PT$

$$P(X+T) = \begin{bmatrix} m & n \\ p & q \end{bmatrix}\begin{bmatrix} x+r & y+s \\ z+t & w+u \end{bmatrix} = \begin{bmatrix} mx+mr+nz+nt & my+ms+nw+nu \\ px+pr+qz+qt & py+ps+qw+qu \end{bmatrix}$$

$$PX = \begin{bmatrix} mx+nz & my+nw \\ px+qz & py+qw \end{bmatrix} \qquad PT = \begin{bmatrix} mr+nt & ms+nu \\ pr+qt & ps+qu \end{bmatrix}$$

$$PX+PT = \begin{bmatrix} mx+mr+nz+nt & my+ms+nw+nu \\ px+pr+qz+qt & py+ps+qw+qu \end{bmatrix}$$

Check: $P(X+T) = PX + PT$

36) $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ a) $IP = \begin{bmatrix} m & n \\ p & q \end{bmatrix} = P$ $PI = \begin{bmatrix} m & n \\ p & q \end{bmatrix} = P$

$IX = \begin{bmatrix} x & y \\ z & w \end{bmatrix} = X$ b) $IT = T$

c) Called identity because it leaves matrices alone – they keep their identity – after multiplication.

3 (48)

$$P = \begin{array}{c} \text{Card.} \quad \text{Finches} \\ \begin{bmatrix} 25 & 31 \\ 20 & 35 \\ 22 & 29 \\ 36 & 20 \end{bmatrix} \end{array} \qquad Q = \begin{array}{c} \text{Sun} \quad \text{Corn} \quad \text{millet} \\ \begin{bmatrix} 32 & 10 & 30 \\ 10 & 18 & 27 \end{bmatrix} \end{array} \qquad R = \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \end{array}$$

a) $QR = \begin{bmatrix} 146 \\ 93 \end{bmatrix}$ Cost per bird per season, for 2 species of birds

$$P(QR) = \begin{bmatrix} 25 & 31 \\ 20 & 35 \\ 22 & 29 \\ 36 & 20 \end{bmatrix}\begin{bmatrix} 146 \\ 93 \end{bmatrix} = \begin{bmatrix} 6533 \\ 6175 \\ 5909 \\ 7116 \end{bmatrix}$$

(Same as $(PQ)R$)