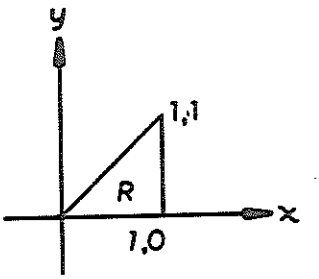
SOLUTIONS

SECTION 15.2

15.2.1 $\displaystyle\int_0^1 \int_y^1 e^{x^2}\,dx\,dy$

$$= \int_0^1 \int_0^x e^{x^2}\,dy\,dx$$

$$= \int_0^1 xe^{x^2}\,dx = \frac{1}{2}(e-1)$$

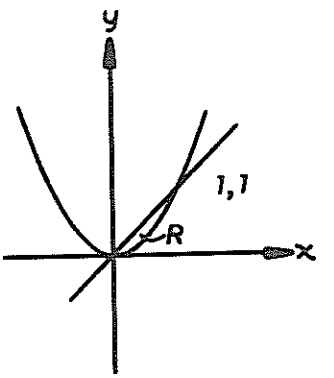


15.2.2 $\displaystyle\int_0^1 \int_{x^2}^x (x^2+y^2)\,dy\,dx$

$$= \int_0^1 \int_y^{\sqrt{y}} (x^2+y^2)\,dx\,dy$$

$$= \int_0^1 \left(\frac{y^{3/2}}{3} + y^{5/2} - \frac{4y^3}{3}\right) dy$$

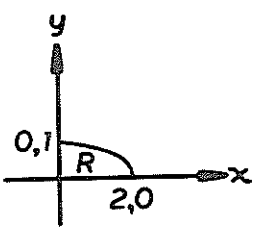
$$= \frac{3}{35}$$



15.2.3 $\displaystyle\int_0^1 \int_0^{2\sqrt{1-y^2}} x\,dx\,dy$

$$= \int_0^2 \int_0^{\sqrt{1-(x^2/4)}} x\,dy\,dx$$

$$= \int_0^2 x\sqrt{1-\frac{x^2}{4}}\,dx = \frac{4}{3}$$

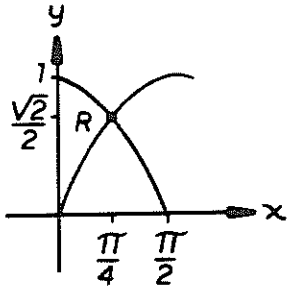


15.2.4 $\displaystyle\int_1^2 \int_0^{\sqrt{x}} y \ln x^2 dx = \int_1^2 x \ln x\,dx = 2\ln 2 - \frac{3}{4}$

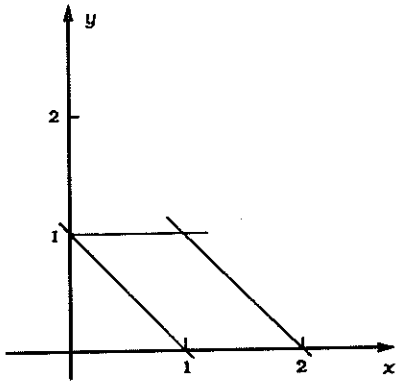
15.2.5 $\displaystyle\int_0^1 \int_{2y}^2 \cos x^2 dx\,dy = \int_0^2 \int_0^{x/2} \cos x^2 dy\,dx = \frac{1}{2}\int_0^2 x\cos x^2 dx = \frac{1}{4}\sin 4$

15.2.6 $\displaystyle\int_0^1 \int_0^x y\sqrt{x^2+y^2}\,dy\,dx = \int_0^1 \frac{2\sqrt{2}-1}{3}x^3 dx = \frac{2\sqrt{2}-1}{12}$

15.2.7
$$\int_0^{\pi/4}\int_{\sin x}^{\cos x} f(x,y)\,dy\,dx = \int_0^{\sqrt{2}/2}\int_0^{\sin^{-1} y} f(x,y)\,dx\,dy + \int_{\sqrt{2}/2}^{1}\int_0^{\cos^{-1} y} f(x,y)\,dx\,dy$$



15.2.8
$$\int_0^1\int_{1-y}^{2-y} f(x,y)\,dx\,dy = \int_0^1\int_{1-x}^{1} f(x,y)\,dy\,dx + \int_1^2\int_0^{2-x} f(x,y)\,dy\,dx$$



15.2.9 $A = \displaystyle\int_0^1\int_{x^2}^{\sqrt{x}} dy\,dx = \int_0^1 (\sqrt{x} - x^2)dx = \frac{1}{3}$

15.2.10 $A = \displaystyle\int_0^2\int_{-y}^{y-y^2} dx\,dy = \int_0^2 (2y - y^2)dy = \frac{4}{3}$

15.2.11 $V = \displaystyle\int_0^2\int_{x^2-x}^{x} (x+1)dy\,dx = \int_0^2 (2x + x^2 - x^3)dx = \frac{8}{3}$

15.2.12 $V = \displaystyle\int_0^4\int_0^{\sqrt{4y}} \frac{1}{2}(2 - x + y)dx\,dy = \frac{1}{2}\int_0^4 (2y^{3/2} - 2y + 4y^{1/2})dy = \frac{232}{15}$

15.2.13 $V = \displaystyle\int_0^4\int_0^{y/2} (4 - x^2)dx\,dy = \int_0^4 \left(2y - \frac{y^3}{24}\right)dy = \frac{40}{3}$

15.2.14 $V = \displaystyle\int_0^2\int_{x^2-x+1}^{x+1} (x+1)dy\,dx = \int_0^2 (2x + x^2 - x^3)dx = \frac{8}{3}$

15.2.15 $V = \displaystyle\int_0^2\int_{x^2}^{2x} (x^2 + y^2)dy\,dx = \int_0^2 \left(\frac{14}{3}x^3 - x^4 - \frac{1}{3}x^6\right)dx = \frac{216}{35}$

15.2.16 $V = \displaystyle\int_0^2 \int_0^y (4 - y^2)dx\,dy = \int_0^2 (4y - y^3)dy = 4$

15.2.17 $V = \displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} y\,dy\,dx = \frac{1}{2}\int_0^2 (4 - x^2)dx = \frac{8}{3}$

15.2.18 $V = 8\displaystyle\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1 - y^2}\,dx\,dy = 8\int_0^1 (1 - y^2)dy = \frac{16}{3}$

15.2.19 6.9684

15.2.20 $5 - \sin(5)$

15.2.21 $\dfrac{8}{35}$

15.2.22 $-\dfrac{\pi}{3}$

15.2.23 $\dfrac{-4\sin\left(\dfrac{\pi^2}{4}\right) + \pi^2}{8}$