HW #6 §9.5

#42, 52, [illegible], 59, 60, 64

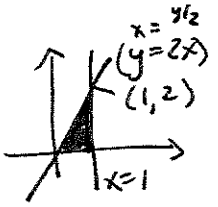
42] $\iint_R 2x^3 e^{x^2y}\,dx\,dy \qquad 0\le x\le 1,\ 0\le y\le 1$

$= \int_0^1 \int_0^1 2x^3 e^{x^2y}\,dy\,dx \quad \begin{pmatrix} u = x^2y \\ du = x^2\,dy \end{pmatrix} = \int_0^1 2x \int_0^1 x^2 e^{x^2y}\,dy\,dx$

$= \int_0^1 2x\, e^{x^2y}\Big|_0^1\,dx = \int_0^1 (2xe^{x^2} - 2x)\,dx = e^{x^2} - \frac{2x^2}{2}\Big|_0^1 = e-1-1$

$= e-2$

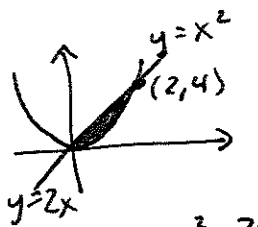
52] $\int_0^2 \int_{y/2}^1 e^{x^2}\,dx\,dy =$



$= \int_0^1 \int_0^{2x} e^{x^2}\,dy\,dx = \int_0^1 ye^{x^2}\Big|_0^{2x}\,dx$

$= \int_0^1 2xe^{x^2}\,dx = e^{x^2}\Big|_0^1 = e-1$

59] $\iint_R x^3 y\,dy\,dx \quad R$ bounded by $y=x^2,\ y=2x$



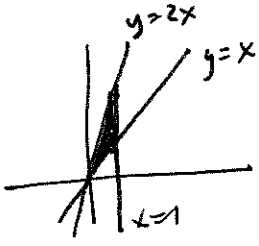
$= \int_0^2 \int_{x^2}^{2x} x^3 y\,dy\,dx = \int_0^2 \frac{x^3y^2}{2}\Big|_{x^2}^{2x}\,dx$

$x^2 = 2x \Rightarrow x=0 \text{ or } x=2$

$= \int_0^2 (2x^5 - \frac{x^7}{2})\,dx = \frac{2x^6}{6} - \frac{x^8}{16}\Big|_0^2 = \frac{64}{3} - \frac{256}{16}$

$= \frac{64}{3} - \frac{48}{3} = \frac{16}{3}$

60] $\iint_R x^2y^2\,dx\,dy \quad R$ bounded by $y=x,\ y=2x,\ x=1$



$= \int_0^1 \int_x^{2x} x^2y^2\,dy\,dx = \int_0^1 \frac{x^2y^3}{3}\Big|_x^{2x}\,dx$

$= \int_0^1 (\frac{8x^5}{3} - \frac{x^5}{3})\,dx = \frac{7}{3}\cdot\frac{x^6}{6}\Big|_0^1 = \frac{7}{18}$

64] Find avg. value of $f(x,y) = 5xy + 2y$ over $1\le x\le 4,\ 1\le y\le 2$

$A = (4-1)\cdot(2-1) = 3$

Avg value $= \frac{1}{3}\int_1^2 \int_1^4 5xy + 2y\,dx\,dy = \frac{1}{3}\int_1^2 \frac{5x^2y}{2} + 2xy\Big|_1^4\,dy$

$= \frac{1}{3}\int_1^2 (40y + 8y - \frac{5}{2}y - 2y)\,dy = \frac{1}{3}\int_1^2 \frac{87}{2}y\,dy$

$= \frac{1}{3}\cdot\frac{87y^2}{4}\Big|_1^2 = \frac{87}{3} - \frac{87}{12}$